Eur. Phys. J. B **42**, 63–83 (2004)
DOI: 10.1140/epjb/e2004-00358-7

THE EUROPEAN PHYSICAL JOURNAL B

The trion: two electrons plus one hole versus one electron plus one exciton

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Received 5 May 2004
Published online 26 November 2004 –

Abstract. We first show that, for problems dealing with trions, it is totally hopeless to use the standard many-body description in terms of electrons and holes and its associated Feynman diagrams. We then show how, by using a description as electrons interacting with excitons, we can obtain the trion absorption through far simpler electron-exciton diagrams. These diagrams are indeed novel because, for excitons being not exact bosons, standard procedures designed to deal with interacting true fermions or true bosons cannot be used. A new many-body formalism is in fact necessary to establish the validity of these electron-exciton diagrams. It relies on the "commutation technique" we recently developed to treat interaction with composite bosons. This technique generates a scattering associated to Coulomb processes between electrons and excitons, without electron exchange, and a "scattering" associated to electron exchange inside the electron-exciton pairs, without Coulomb process — this Pauli scattering being the original part of our new many-body theory.

PACS. 71.35.-y Excitons and related phenomena

1 Introduction

While the physics of excitons and electron-hole plasma has been a subject of great interest in the 60's and 70's, the physics of trions [1,2], i.e., excitons bound to an electron or a hole, developed recently only: The exciton being neutral, its binding to a carrier is quite weak compared to the electron-hole (e-h) binding. However, due to the development of nanostructures, it should be now possible to experimentally study these trions because, as all binding energies are enhanced by the reduction of dimensionality, trions, hard to see in bulk samples, should appear as line well below the exciton line in the absorption spectra of doped semiconductor quantum wells (see for instance Ref. [3–9]).

From a theoretical point of view [10–19], the study of these trions still faces major difficulties: (i) Being the eigenstates of two electrons and one hole (e-e-h) – or two holes and one electron – in Coulomb interaction, their energies and wave functions are not analytically known; (ii) because these trions are bound states, they cannot be reached from a (finite) perturbative approach; (iii) while many-body procedures have been developed in the 60's to treat interactions between fermions (or bosons), we show that they are completely inappropriate for bound states resulting from the *exact* summation of all Coulomb processes between more than two fermions, as in the trion case.

With respect to this last point, a good idea can be to bind one electron to the hole for the trion to appear as a two-body system: one electron interacting with one exciton [18]. However, since electrons are indistinguishable, the electron which is bound to the hole to make the exciton, is a priori arbitrary. Moreover, as another aspect of this arbitrariness, the exciton is not an exact boson so that standard many-body procedures, designed for exact fermions or exact bosons, cannot be used for this interacting electron-exciton (e-X) system.

In spite of these difficulties, the description of a trion as an electron interacting with an exciton is physically appealing. In particular, it allows to immediately realize that, as the exciton is neutral, its attraction is very weak, so that the trion binding has to be much weaker than the exciton binding. It is thus worth finding a way to overcome the difficulties this e-X description raises.

In order to use it, we first have to identify a scattering for the e-X Coulomb interaction, although, due to the composite nature of the exciton, there is no way to extract it from the Coulomb potential between individual carriers. In addition, we must find a way to take care of the electron indistinguishability when constructing the exciton. This indistinguishability somehow adds novel exchange "scatterings" to the more standard e-X Coulomb scatterings,

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with specific rules which have to be determined — and which are the novel part of our e-X diagrams.

This paper in fact deals with the simplest problem on trions, namely the absorption of one photon in the presence of *one* carrier. The photon creating one e-h pair, we will first consider it in the framework of an e-e-h system, and show that the corresponding response function, written with standard e-h diagrams, is so complicated that there is no hope to identify and sum up all the Coulomb processes responsible for trions.

We will then show that this response function appears very simply if we bind one electron to the hole and have this exciton scattered by its interactions with the additional electron. The spin conservation of the semiconductor-photon interaction leads to differentiate absorptions in which the spins of the photocreated and initial electrons are identical or different. We will show that electron exchange enters the photon absorption *explicitly* in the first case only, i.e., when triplet trions are the only ones created, in spite of the fact that triplet trions can also be created when the two electrons have different spins. We will establish the rules for these e-X diagrams, using our commutation technique for excitons interacting with electrons [18]. It allows to calculate the response function to a photon field at any order in the e-X interaction, while keeping the composite nature of the exciton, i.e., the fact that the exciton can be made with any of the two electrons.

This first paper allows to establish the problem of one exciton interacting with one electron on a firm basis, in order to possibly face a much harder one, namely one exciton interacting with N electrons, as for photon absorption in the presence of a Fermi sea [11,14]. Such an absorption has been considered long ago by Combescot and Nozières [20]. In their work, the spin degree of freedom of the electrons has been dropped as well as the electron-electron (e-e) repulsion. While the first simplification physically corresponds to have one kind of spin only in the problem — as for a σ_- photon absorbed by a quantum well having a $(+1/2)$ polarized Fermi sea so that $(S_z = 1)$ triplet trions are the only ones possibly formed —, the second simplification is dramatic: Once it is made, the trion physics is irretrievably lost. Indeed, the e-e repulsion partly compensates the e-h attraction, making the trion binding energy much weaker than the exciton one.

Let us stress that this crucial effect of the e-e interaction cannot be included through a screening of the e-h attraction, as suggested by Hawrylak in his 2D extension [11] of Combescot-Nozières's work. Indeed, such a screening in the same way reduces the e-h attraction responsible for the exciton binding, so that the trion binding is then weak but *equal* to the exciton one, not smaller as it should. This screening procedure in fact generates one energy level, not two, this level being possibly occupied by two electrons with different spins. This procedure thus misses the whole trion physics, with an electron very weakly bound, compared to the other.

To get a trion binding energy much weaker than the exciton one, we have to include the e-e interaction independently from the e-h attraction, not just through a screening. The trouble is that, within the many-body procedures at hand up to now, there is no hope to add this e-e repulsion to the set of already complicated e-h processes which, summed up, give rise to the so called "Fermi edge singularities". In this paper, we explicitly show that the e-h Feynman diagrams are already inappropriate to describe just one trion. A new many-body formalism is thus highly needed if we want to describe the trion absorption change with doping observed experimentally. Its presentation is in fact *the underlying purpose of this work*, with a particular attention paid to the interplay between the somewhat normal e-X direct Coulomb scattering and the far more subtle electron exchange inside the e-X pair.

The paper is organized as follows:

In Section 2, we briefly recall the usual procedure to calculate photon absorption and we try to calculate it in the case of trion formation, using the standard many-body procedure which leads to expand it through the well known e-h Feynman diagrams. We show that, even in the simple case of a photon creating an electron with a spin different from the initial one, the summation of these diagrams, with all possible Coulomb processes between e-e-h, is totally hopeless.

In Section 3, we reconsider our description of a trion as an electron interacting with an exciton, the presentation given here being more direct than the one of our previous works [17,18].

In Section 4, we calculate the trion response function in terms of these interacting e-X pairs. We show that it appears as a sum of e-X ladder diagrams, with possibly *one* — but no more than one — electron exchange between e and X, if — and only if — the spins of the photocreated and initial electrons are the same. This result, which can be surprising at first since, at each e-X scattering, the exciton can a priori be constructed with any of the two electrons, physically comes from the fact that two exchanges reduce to an identity.

2 The trion as two electrons plus one hole

A quite direct way to reach the trion is to look at the photon absorption when the semiconductor has one electron already present in the sample. Let us first briefly recall how photon absorption is usually obtained and how it appears in the case of exciton formation, since its comparison with trion will be of great help.

2.1 Standard procedure to calculate photon absorption

The linear absorption of a photon field is given by the Fermi golden rule,

$$\frac{2\pi}{\hbar}\sum_F |\langle F|W^\dagger|I\rangle|^2\,\delta(\mathbb{E}_F-\mathbb{E}_I) = -\frac{2}{\hbar}\,\mathrm{Im}S, \qquad (2.1)$$

where, due to $1/(x+i\eta) = \mathcal{P}(1/x) - i\pi\delta(x)$, the response function S reads

$$S = \langle I|W \frac{1}{\mathbb{E}_I - \hat{H} + i\eta} W^\dagger|I\rangle. \quad (2.2)$$

$\hat{H}$ is the Hamiltonian of the uncoupled matter-photon system, $\hat{H} = H + \omega_p \alpha^\dagger \alpha$, with H being the matter Hamiltonian and $\alpha^\dagger$ the creation operator of $(\omega_p, \mathbf{Q}_p)$ photons. The matter-photon coupling $W^\dagger$ reads $W^\dagger = U^\dagger \alpha$, where for circularly polarized photons $(\pm)$ in quantum wells, we have, due to momentum conservation,

$$U^\dagger_\pm(\mathbf{Q}_p) = \lambda^* \sum_{\mathbf{p}} a^\dagger_{\mathbf{p}+\alpha_e\mathbf{Q}_p,\mp} b^\dagger_{-\mathbf{p}+\alpha_h\mathbf{Q}_p,\pm},$$
$$\alpha_e = 1 - \alpha_h = \frac{m_e}{m_e + m_h}. \quad (2.3)$$

$a^\dagger_{\mathbf{p},\mp}$ creates an electron with momentum $\mathbf{p}$ and spin $\mp 1/2$, while $b^\dagger_{\mathbf{p},\pm}$ creates a hole with spin $\pm 3/2$. The reason for this splitting of $\mathbf{Q}_p$ between (e, h) will become apparent below.

For N_p photons and a matter initial state $|i\rangle$ with energy $\mathbb{E}_i$, the initial state in equation (2.2) reads $|I\rangle = |N_p\rangle \otimes |i\rangle$, the initial energy $\mathbb{E}_I$ being $N_p\omega_p + \mathbb{E}_i$. This leads to

$$S_\pm = N_p \langle i|U_\pm(\mathbf{Q}_p) \frac{1}{\omega_p + \mathbb{E}_i - H + i\eta} U^\dagger_\pm(\mathbf{Q}_p)|i\rangle. \quad (2.4)$$

In equation (2.4), H acts on the photocreated e-h pair plus the initial carriers. Many-body effects between them follow from the identity

$$\frac{1}{a-H} = \frac{1}{a-H_0} + \frac{1}{a-H} V \frac{1}{a-H_0}, \quad (2.5)$$

valid for $H = H_0 + V$, which can be iterated as

$$\frac{1}{a-H} = \frac{1}{a-H_0} + \frac{1}{a-H_0} V \frac{1}{a-H_0} + \cdots \quad (2.6)$$

In the case of semiconductors, the free part H_0 reads

$$H_0 = \sum_{\mathbf{k},s} \epsilon^{(e)}_{\mathbf{k}} a^\dagger_{\mathbf{k},s} a_{\mathbf{k},s} + \sum_{\mathbf{k},m} \epsilon^{(h)}_{\mathbf{k}} b^\dagger_{\mathbf{k},m} b_{\mathbf{k},m},$$
$$\epsilon^{(e,h)}_{\mathbf{k}} = \frac{\hbar^2\mathbf{k}^2}{2m_{e,h}}, \quad (2.7)$$

while V is the Coulomb potential between carriers,

$$V = \frac{1}{2}\sum_{\mathbf{q}\neq\mathbf{0}} V_{\mathbf{q}} \Bigg[\sum_{\mathbf{k},\mathbf{k}',s,s'} a^\dagger_{\mathbf{k}+\mathbf{q},s} a^\dagger_{\mathbf{k}'-\mathbf{q},s'} a_{\mathbf{k}',s'} a_{\mathbf{k},s}$$
$$+ \sum_{\mathbf{k},\mathbf{k}',m,m'} b^\dagger_{\mathbf{k}+\mathbf{q},m} b^\dagger_{\mathbf{k}'-\mathbf{q},m'} b_{\mathbf{k}',m'} b_{\mathbf{k},m}$$
$$-2 \sum_{\mathbf{k},\mathbf{k}',s,m'} a^\dagger_{\mathbf{k}+\mathbf{q},s} b^\dagger_{\mathbf{k}'-\mathbf{q},m'} b_{\mathbf{k}',m'} a_{\mathbf{k},s} \Bigg], \quad (2.8)$$

with $V_{\mathbf{q}} = 2\pi e^2/\Omega q$ in 2D, Ω being the sample volume.

2.2 Photon absorption with exciton formation, using electron-hole diagrams

For σ_+ photons absorbed in an *empty* well, the response function given in equation (2.4) reads, due to equation (2.3),

$$S_X = N_p|\lambda|^2 \sum_{\mathbf{p}',\mathbf{p}} \langle v| b_{-\mathbf{p}'+\alpha_h\mathbf{Q}_p,+}\, a_{\mathbf{p}'+\alpha_e\mathbf{Q}_p,-}$$
$$\times \frac{1}{\omega_p - H + i\eta} a^\dagger_{\mathbf{p}+\alpha_e\mathbf{Q}_p,-} b^\dagger_{-\mathbf{p}+\alpha_h\mathbf{Q}_p,+}|v\rangle, \quad (2.9)$$

$|v\rangle$ being the e-h vacuum state with an energy chosen as zero. S_X is calculated using the expansion (2.6). Due to our Q_p splitting, $\epsilon^{(e)}_{\mathbf{p}+\alpha_e\mathbf{Q}_p} + \epsilon^{(h)}_{-\mathbf{p}+\alpha_h\mathbf{Q}_p} = \epsilon^{(X)}_{\mathbf{p}} + \mathcal{E}^{(X)}_{\mathbf{Q}_p}$, with

$$\epsilon^{(X)}_{\mathbf{p}} = \frac{\hbar^2\mathbf{p}^2}{2(m_e^{-1} + m_h^{-1})^{-1}}$$
$$\mathcal{E}^{(X)}_{\mathbf{Q}} = \frac{\hbar^2\mathbf{Q}^2}{2(m_e + m_h)}, \quad (2.10)$$

so that the zero order term reduces to

$$S^{(0)}_X = N_p|\lambda|^2 \sum_{\mathbf{p}} \frac{1}{\omega_p - \epsilon^{(X)}_{\mathbf{p}} - \mathcal{E}^{(X)}_{\mathbf{Q}_p} + i\eta}$$
$$= N_p|\lambda|^2 \sum_{\mathbf{p}} G^{(eh)}(\omega_p, \mathbf{Q}_p; \mathbf{p}), \quad (2.11)$$

which can be seen as a definition of the free e-h propagator $G^{(eh)}(\omega_p, \mathbf{Q}_p; \mathbf{p})$.

The first order term in Coulomb interaction appears as

$$S^{(1)}_X = N_p|\lambda|^2 \sum_{\mathbf{p}',\mathbf{p}} G^{(eh)}(\omega_p, \mathbf{Q}_p; \mathbf{p}')$$
$$\times V_{\mathbf{p}'-\mathbf{p}}\, G^{(eh)}(\omega_p, \mathbf{Q}_p; \mathbf{p}). \quad (2.12)$$

The second order term contains three $G^{(eh)}$ and two Coulomb potentials; and so on... The exciton response function thus corresponds to the well known set of e-h ladder diagrams [21] shown in Figure 1, $G^{(eh)}$ being nothing but

$$G^{(eh)}(\omega_p, \mathbf{Q}_p; \mathbf{p}) = \int \frac{id\omega}{2\pi}\, g^{(e)}(\omega + \omega_p, \mathbf{p} + \alpha_e\mathbf{Q}_p)$$
$$\times\, g^{(h)}(-\omega, -\mathbf{p} + \alpha_h\mathbf{Q}_p), \quad (2.13)$$

where $g^{(e)}(\omega, \mathbf{k}) = [\omega - \epsilon^{(e)}_{\mathbf{k}} + i\eta]^{-1}$ and $g^{(h)}(\omega, \mathbf{k}) = [\omega - \epsilon^{(h)}_{\mathbf{k}} + i\eta]^{-1}$ are the free electron and free hole propagators for empty bands.

2.3 Photon absorption with trion formation, using electron-hole diagrams

To get a X^- trion, the semiconductor initial state must have one electron. If $\mathbf{k}_i$ and s_i are its momentum and spin, this initial state reads $|i\rangle = a^\dagger_{\mathbf{k}_i,s_i}|v\rangle$, with $\mathbb{E}_i = \epsilon^{(e)}_{\mathbf{k}_i}$. While

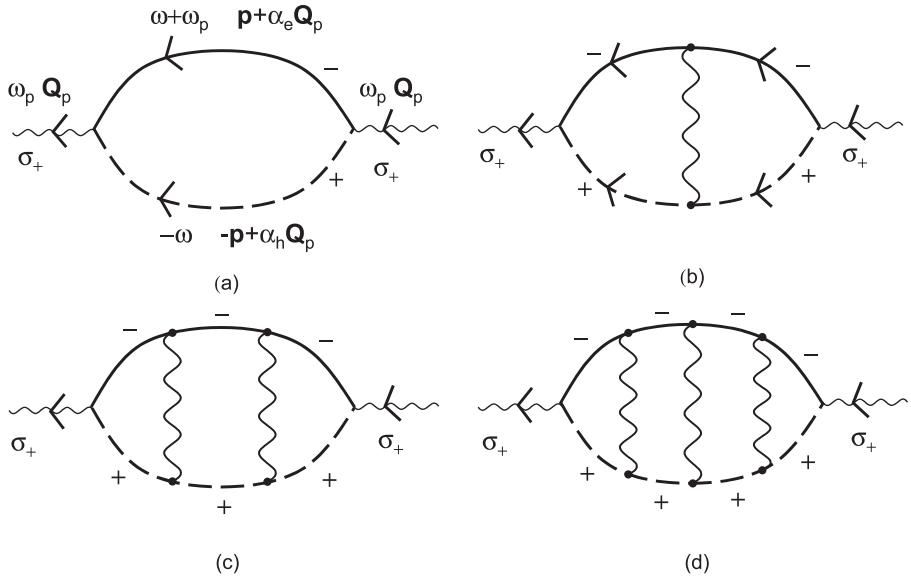

Fig. 1. Photon absorption with exciton formation: well-known set of e-h ladder diagrams with 0, 1, 2, 3... e-h Coulomb interactions: (a), (b), (c), (d) respectively. Solid line: electron. Dashed line: hole. Wavy line: Coulomb interaction between electron and hole. In a well, a σ_+ photon creates a hole with momentum (+3/2), noted +, and an electron with spin (−1/2), noted −.

the photon polarization is unimportant for an empty semiconductor, it is crucial when electrons are present: In a quantum well, if the spins of the photocreated and initial electrons are different, the hole can only recombine with the photocreated electron, while it can recombine with any electron if the spins are identical. Consequently, the response function crucially depends on the electron spins. Let us start with the simplest case.

2.3.1 Photocreated electron with spin different from the initial one

This happens when a σ_+ photon is absorbed in a quantum well having a $s_i = +1/2$ electron. Using equations (2.3–4), the zero order term of the response function reads

$$\tilde{S}^{(0)}_{\neq} = N_p|\lambda|^2 \sum_{\mathbf{p}',\mathbf{p}} \langle v| a_{\mathbf{k}_i,+} b_{-\mathbf{p}'+\alpha_h\mathbf{Q}_p,+} a_{\mathbf{p}'+\alpha_e\mathbf{Q}_p,-} \times \left(\frac{1}{\omega_p + \epsilon^{(e)}_{\mathbf{k}_i} - H_0 + i\eta} \right) a^\dagger_{\mathbf{p}+\alpha_e\mathbf{Q}_p,-}\, b^\dagger_{-\mathbf{p}+\alpha_h\mathbf{Q}_p,+}\, a^\dagger_{\mathbf{k}_i,+}|v\rangle, \tag{2.14}$$

which is nothing but the exciton zero order response function $S^{(0)}_X$, so that $\tilde{S}^{(0)}_{\neq}$ corresponds to the diagram of Figure 1a. If we now turn to the first order term, it is given by

$$\tilde{S}^{(1)}_{\neq} = N_p|\lambda|^2 \sum_{\mathbf{p}',\mathbf{p}} G^{(eh)}(\omega_p, \mathbf{Q}_p; \mathbf{p}') G^{(eh)}(\omega_p, \mathbf{Q}_p; \mathbf{p}) \times \langle v| a_{\mathbf{k}_i,+}\, b_{-\mathbf{p}'+\alpha_h\mathbf{Q}_p,+}\, a_{\mathbf{p}'+\alpha_e\mathbf{Q}_p,-} \times V\, a^\dagger_{\mathbf{p}+\alpha_e\mathbf{Q}_p,-}\, b^\dagger_{-\mathbf{p}+\alpha_h\mathbf{Q}_p,+}\, a^\dagger_{\mathbf{k}_i,+}|v\rangle, \tag{2.15}$$

where V is the Coulomb potential of equation (2.8). As this Coulomb potential only contains $\mathbf{q} \neq \mathbf{0}$ excitations, the above matrix element differs from 0 for $\mathbf{p}' = \mathbf{p}+\mathbf{q}$ only: This first order term is thus equal to $S^{(1)}_X$ and corresponds to the ladder diagram of Figure 1b.

The higher order terms are not as simple: When more than one Coulomb excitation take place, in addition to processes in which the photocreated electron is scattered several times by the hole, other processes involving the initial electron become possible: In addition to the exciton ladder diagrams of Figure 1c at second order in V and Figure 1d at third order, we also have the 4 diagrams of Figure 2 at second order in V and the 20 diagrams of Figure 3 at third order. Note that, since the (+1/2) electron "band" contains one electron only, these diagrams have one conduction-hole line only, without any possible scattering, i.e., one electron line going from left to right.

As the X^- trion corresponds to the *bound* state of e-e-h resulting from their Coulomb interactions, the Coulomb potential has to be taken *exactly* into account i.e., included at all orders, to possibly generate bound state poles in the response function. In view of the third order processes shown in Figure 3, it is obviously hopeless to draw the diagrams for all these Coulomb interactions, at any order in V, and to sum them up to get the trion.

2.3.2 Photocreated electron with spin identical to the initial one

The situation is worse when the photocreated and initial electrons have the same spin, because the hole can now recombine with any of the two electrons. While the zero order term, given in equation (2.14), with $a^\dagger_{\mathbf{k}_i,+}$ replaced

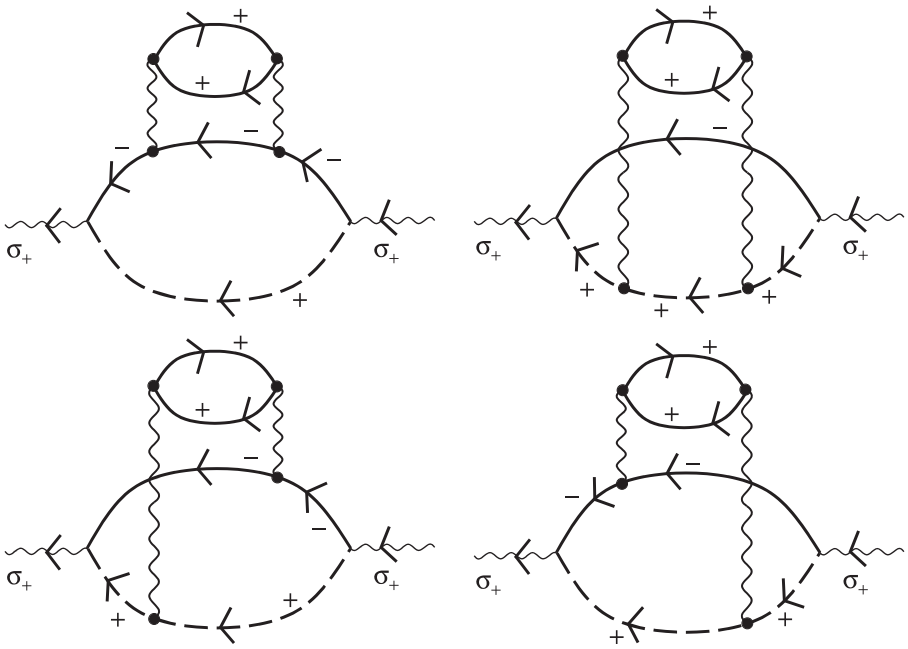

Fig. 2. Absorption of a σ_+ photon with trion formation, in the case of photocreated and initial electrons having different spins: second order in Coulomb interaction. The diagrams of this figure have to be added to the one of Figure 1c.

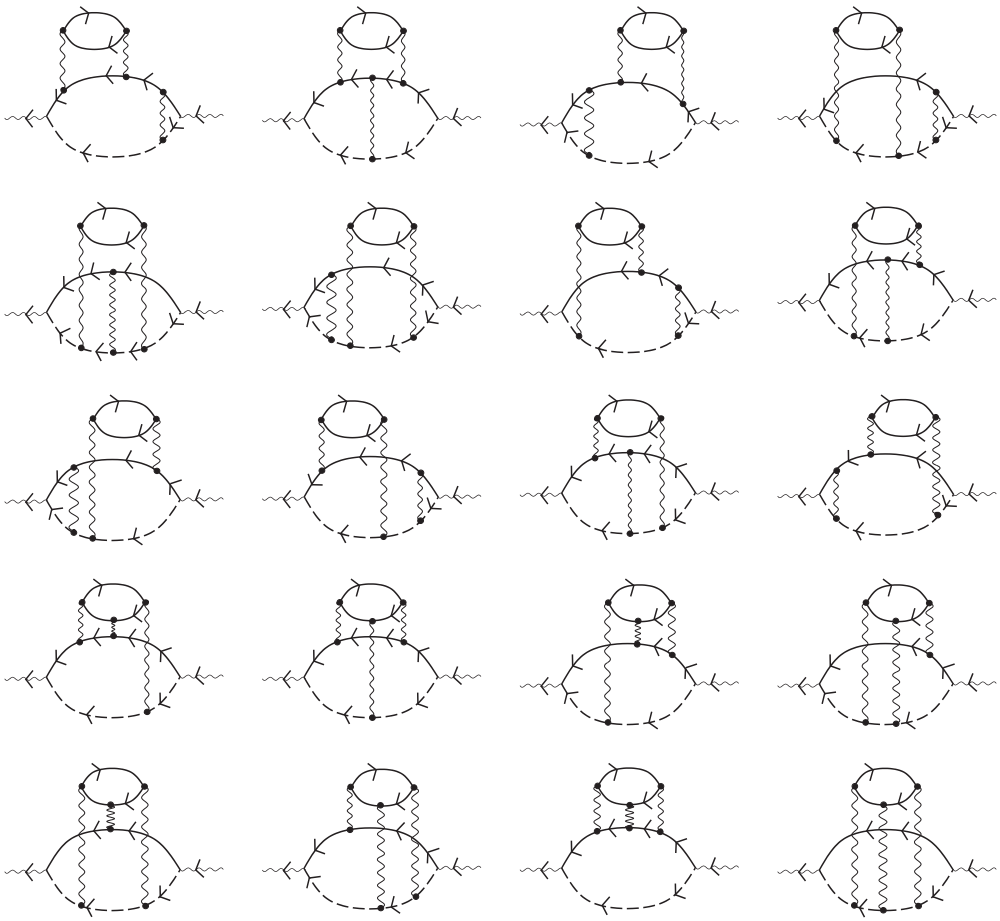

Fig. 3. Same as Figure 2, with three Coulomb interactions. The diagrams of this figure have to be added to the one of Figure 1d.

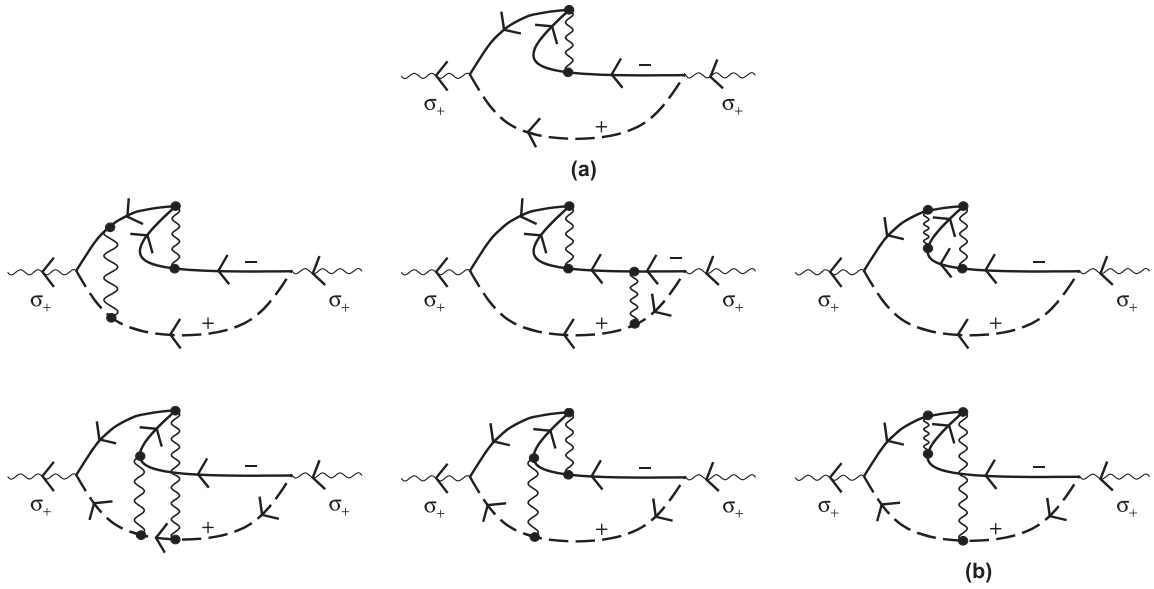

Fig. 4. Absorption of a σ_+ photon with trion formation, in the case of photocreated and initial electrons having the same spin, at first order (a) and second order (b) in Coulomb interaction. The diagrams of this figure, which correspond to exchanges between the photocreated and initial electrons, have to be added to the diagrams of Figures 1b, 1c, 2.

by $a^\dagger_{\mathbf{k}_i,-}$, stays essentially unchanged, $\mathbf{p}$ being just different from $\mathbf{k}_i - \alpha_e \mathbf{Q}_p$, new diagrams with exchange processes between the two electrons appear at higher orders. From equation (2.15) with $a^\dagger_{\mathbf{k}_i,+}$ replaced by $a^\dagger_{\mathbf{k}_i,-}$, we see that, beside $\mathbf{p}' = \mathbf{p} + \mathbf{q}$ which gives the first order ladder diagram of Figure 1b, we can also have $\mathbf{p} = \mathbf{p}' = \mathbf{k}_i - \mathbf{q}$ which gives the first order exchange diagram of Figure 4a. Similarly, beside the second order direct diagrams of $\tilde{S}^{(2)}_{\neq}$, we also have the six diagrams of Figure 4b which result from exchange processes between the photocreated and the initial electrons; and so on...

This leads us to conclude that the representation of a trion as e-e-h, and its associated standard e-h Feynman diagrams, are completely inappropriate.

The description of a trion as an electron interacting with an exciton is actually far better. Let us first recall the main steps of the many-body procedure on which it is based and which allows an exact treatment of the tricky part of this description, namely the electron indistinguishability. This procedure ultimately leads to represent the trion through novel e-X diagrams, with specific rules, not easy to guess at first for the new Pauli "scatterings" coming from pure exchange, i.e., exchange without Coulomb.

3 The trion as one electron plus one exciton

By considering the trion as an electron interacting with an exciton, we tend to put the trion and the exciton on equal footing — with the hole of the exciton just replaced by an exciton. This is actually misleading because, due to the electron indistinguishability, the trion is definitely far more subtle than the exciton. In order to grasp the deep differences between them, it can be useful to briefly recall a few well known results on exciton. This will allow us to settle important notations, also useful for trion.

3.1 A few results on exciton

3.1.1 First quantization

The exciton can be seen as a two-body object, made of one electron $(m_e, \mathbf{r}_e)$ and one hole $(m_h, \mathbf{r}_h)$. If we extract its center of mass,

$$M_X = m_e + m_h \qquad \mathbf{R}_X = (m_e \mathbf{r}_e + m_h \mathbf{r}_h)/M_X, \quad (3.1)$$

we are left with its relative motion, characterized by

$$\mu_X^{-1} = m_e^{-1} + m_h^{-1} \qquad \mathbf{r} = \mathbf{r}_e - \mathbf{r}_h. \quad (3.2)$$

The exciton Hamiltonian reads

$$H_X = \frac{\mathbf{p}_e^2}{2m_e} + \frac{\mathbf{p}_h^2}{2m_h} - \frac{e^2}{|\mathbf{r}_e - \mathbf{r}_h|} = \frac{\mathbf{P}_X^2}{2M_X} + h_X, \quad (3.3)$$

where $h_X = h_X^{(0)} - v(\mathbf{r})$ is the exciton relative motion Hamiltonian composed of a free-particle part $h_X^{(0)} = \mathbf{p}_\mathbf{r}^2/2\mu_X$ and a Coulomb attraction $-v(\mathbf{r}) = -e^2/r$.

The eigenstates of the relative motion free part are the plane waves $|\mathbf{p}\rangle$ of energies $\epsilon_\mathbf{p}^{(X)}$ given in equation (2.10): $(h_X^{(0)} - \epsilon_\mathbf{p}^{(X)})|\mathbf{p}\rangle = 0$. They are such that $\langle \mathbf{p}'|\mathbf{p}\rangle = \delta_{\mathbf{p}',\mathbf{p}}$, their closure relation being $\sum_\mathbf{p} |\mathbf{p}\rangle\langle\mathbf{p}| = I$.

The eigenstates $|\nu\rangle$ of the full relative motion Hamiltonian, $(h_X - \varepsilon_\nu^{(X)})|\nu\rangle = 0$, are made of bound *and* diffusive states. They are such that $\langle \nu'|\nu\rangle = \delta_{\nu',\nu}$, their closure relation being $\sum_\nu |\nu\rangle\langle\nu| = I$.

In terms of $|\mathbf{p}\rangle$, these $|\nu\rangle$ states read $|\nu\rangle = \sum_{\mathbf{p}} |\mathbf{p}\rangle\langle\mathbf{p}|\nu\rangle$, so that the projection over $\langle\mathbf{p}|$ of the Schrödinger equation for $|\nu\rangle$ leads to

$$(\epsilon_{\mathbf{p}}^{(X)} - \varepsilon_{\nu}^{(X)})\langle\mathbf{p}|\nu\rangle - \sum_{\mathbf{p}'}\langle\mathbf{p}|v(\mathbf{r})|\mathbf{p}'\rangle\,\langle\mathbf{p}'|\nu\rangle = 0, \qquad (3.4)$$

$$\langle\mathbf{p}|v(\mathbf{r})|\mathbf{p}'\rangle = V_{\mathbf{p}'-\mathbf{p}}, \qquad (3.5)$$

where $V_{\mathbf{q}}$ is the Fourier transform of the Coulomb potential $v(\mathbf{r})$ appearing in h_X.

If we come back to the full exciton Hamiltonian H_X, its eigenstates are the $|\nu,\mathbf{Q}\rangle$'s with wave functions $\langle\mathbf{r}_e,\mathbf{r}_h|\nu,\mathbf{Q}\rangle = \langle\mathbf{r}|\nu\rangle\langle\mathbf{R}_X|\mathbf{Q}\rangle$, where $|\mathbf{Q}\rangle$ is the plane wave of energy $\mathcal{E}_{\mathbf{Q}}^{(X)}$ given in equation (2.10): We do have $(H_X - E_{\nu,\mathbf{Q}}^{(X)})|\nu,\mathbf{Q}\rangle = 0$, with

$$E_{\nu,\mathbf{Q}}^{(X)} = \varepsilon_{\nu}^{(X)} + \mathcal{E}_{\mathbf{Q}}^{(X)}. \qquad (3.6)$$

3.1.2 Second quantization

In second quantization, the semiconductor Hamiltonian has the same form, equations (2.7–8), whatever the e, h numbers. In terms of these free electron and hole creation operators, the creation operator for an exciton $(\nu,\mathbf{Q})$ with electron spin s and hole momentum m reads

$$B_{\nu,\mathbf{Q},s,m}^{\dagger} = \sum_{\mathbf{p}}\langle\mathbf{p}|\nu\rangle\, a_{\mathbf{p}+\alpha_e\mathbf{Q},s}^{\dagger}\, b_{-\mathbf{p}+\alpha_h\mathbf{Q},m}^{\dagger}\,: \qquad (3.7)$$

$(H - E_{\nu,\mathbf{Q}}^{(X)})B_{\nu,\mathbf{Q},s,m}^{\dagger}|v\rangle = 0$ is easy to check from equations (2.7−8; 3.4, 7). With this splitting of the exciton momentum $\mathbf{Q}$ between (e, h), $\mathbf{p}$ is just the momentum of the exciton relative motion. As easy to check from equation (3.7), we also have

$$a_{\mathbf{p}+\alpha_e\mathbf{Q},s}^{\dagger}\, b_{-\mathbf{p}+\alpha_h\mathbf{Q},m}^{\dagger} = \sum_{\nu}\langle\nu|\mathbf{p}\rangle\, B_{\nu,\mathbf{Q},s,m}^{\dagger}. \qquad (3.8)$$

3.2 Trion in first quantization

To simplify the notations, we will here restrict to trions made with two electrons and one hole.

A first difficulty with trions, compared to excitons, arises from spins. They are unimportant for excitons if we neglect "electron-hole exchange", i.e., valence-conduction Coulomb excitations, the exciton energies being then degenerate with respect to (s,m). On the opposite, spins are crucial for trions because they differentiate their states through the parity of the wave function orbital part, with respect to the electron positions. This parity is actually linked to the trion total *electronic* spin: Singlets being odd while triplets are even, their associate orbital wave functions must be even and odd respectively, due to the symmetry principle for the whole fermion wave function.

Another difficulty comes from the appropriate variables to describe the trion. The center of mass $(M_T,\mathbf{R}_T)$ is surely a good variable: For two electrons $(m_e,\mathbf{r}_e,\mathbf{r}_{e'})$ and one hole $(m_h,\mathbf{r}_h)$, it reads

$$\begin{aligned} M_T &= 2m_e + m_h \\ \mathbf{R}_T &= (m_e\mathbf{r}_e + m_e\mathbf{r}_{e'} + m_h\mathbf{r}_h)/M_T. \end{aligned} \qquad (3.9)$$

While for excitons, there is only one variable $\mathbf{r}$ which, along with $\mathbf{R}_X$, forms a good set of variables, i.e., for which $[r_i,p_j] = i\hbar\delta_{ij}$, there are many ways [17] to choose the two other spatial variables which, along with $\mathbf{R}_T$, form a good set for trions. Among them, *the convenient ones for physical understanding* are $\mathbf{r}$, associated to μ_X, defined in equation (3.2), and $\mathbf{u}$, associated to μ_T, defined as

$$\begin{aligned} \mu_T^{-1} &= m_e^{-1} + M_X^{-1} \\ \mathbf{u} &= \mathbf{r}_{e'} - \mathbf{R}_X, \end{aligned} \qquad (3.10)$$

with (M_X,R_X) defined in equation (3.1). $\mathbf{u}$ is the distance between e' and the center of mass of (e,h), while μ_T is the relative motion mass of this e' electron and the (e,h) exciton. Of course, due to the electron indistinguishability, variables as good as $(\mathbf{r},\mathbf{u})$ are

$$\begin{aligned} \mathbf{r}' &= \mathbf{r}_{e'} - \mathbf{r}_h = \mathbf{u} + \alpha_e\mathbf{r} \\ \mathbf{u}' &= \mathbf{r}_e - \mathbf{R}'_X = (1-\alpha_e^2)\mathbf{r} - \alpha_e\mathbf{u}, \end{aligned} \qquad (3.11)$$

with $\mathbf{R}'_X = (m_e\mathbf{r}_{e'} + m_h\mathbf{r}_h)/M_X$. This possible change from $(\mathbf{r},\mathbf{u})$ to $(\mathbf{r}',\mathbf{u}')$, which corresponds to an electron exchange in the trion, is present all over the trion representation in terms of e-X. We will show below how to handle it.

In terms of these variables, the trion Hamiltonian reads

$$\begin{aligned} H_T &= \frac{\mathbf{p}_e^2}{2m_e} + \frac{\mathbf{p}_{e'}^2}{2m_e} + \frac{\mathbf{p}_h^2}{2m_h} \\ &\quad - \frac{e^2}{|\mathbf{r}_e-\mathbf{r}_h|} - \frac{e^2}{|\mathbf{r}_{e'}-\mathbf{r}_h|} + \frac{e^2}{|\mathbf{r}_e-\mathbf{r}'_e|} \\ &= \frac{\mathbf{P}_T^2}{2M_T} + h_T. \end{aligned} \qquad (3.12)$$

Like for excitons, the relative motion Hamiltonian h_T splits as $h_T^{(0)} + w(\mathbf{r},\mathbf{u})$, with a free part $h_T^{(0)} = h_X + \mathbf{p}_{\mathbf{u}}^2/2\mu_T$, made of one exciton plus one effective particle of mass μ_T, and an interaction $w(\mathbf{r},\mathbf{u})$ which corresponds to Coulomb processes between e' and (e,h),

$$\begin{aligned} w(\mathbf{r},\mathbf{u}) &= \frac{e^2}{|\mathbf{r}_{e'}-\mathbf{r}_e|} - \frac{e^2}{|\mathbf{r}_{e'}-\mathbf{r}_h|} \\ &= \frac{e^2}{|\mathbf{u}-\alpha_h\mathbf{r}|} - \frac{e^2}{|\mathbf{u}+\alpha_e\mathbf{r}|}. \end{aligned} \qquad (3.13)$$

The eigenstates of the relative motion free part, $|\nu,\mathbf{p}\rangle$, are such that $h_T^{(0)}|\nu,\mathbf{p}\rangle = (\varepsilon_{\nu}^{(X)} + \epsilon_{\mathbf{p}}^{(eX)})|\nu,\mathbf{p}\rangle$, with

$$\epsilon_{\mathbf{p}}^{(eX)} = \frac{\hbar^2\mathbf{p}^2}{2\mu_T}, \qquad (3.14)$$

and $\langle\mathbf{r},\mathbf{u}|\nu,\mathbf{p}\rangle = \langle\mathbf{r}|\nu\rangle\langle\mathbf{u}|\mathbf{p}\rangle$. They fulfil $\langle\nu',\mathbf{p}'|\nu,\mathbf{p}\rangle = \delta_{\nu',\nu}\delta_{\mathbf{p}',\mathbf{p}}$, while their closure relation reads $\sum_{\nu,\mathbf{p}}|\nu,\mathbf{p}\rangle\langle\nu,\mathbf{p}| = I$.

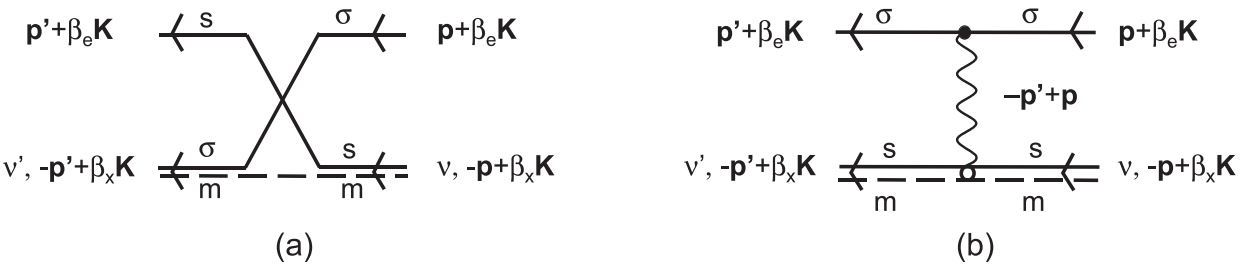

Fig. 5. (a) Exchange parameter $L_{\nu'\mathbf{p}';\nu\mathbf{p}}$. The "in" exciton ν and the "out" exciton ν' are made with different electrons. No Coulomb interaction takes place in this scattering. (b) e-X direct Coulomb scattering $C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}}$. The "in" exciton ν and the "out" exciton ν' are made with the same electron.

Let us call $|\eta\rangle$ the eigenstates of the trion relative motion Hamiltonian, $(h_T - \varepsilon^{(T)}_\eta)|\eta\rangle = 0$. Using them, the eigenstates of the trion full Hamiltonian H_T are the $|\eta, \mathbf{K}\rangle$'s, since $(H_T - E^{(T)}_{\eta,\mathbf{K}})|\eta, \mathbf{K}\rangle = 0$, with

$$E^{(T)}_{\eta,\mathbf{K}} = \varepsilon^{(T)}_\eta + \mathcal{E}^{(T)}_{\mathbf{K}}, \qquad \mathcal{E}^{(T)}_{\mathbf{K}} = \frac{\hbar^2\mathbf{K}^2}{2M_T}, \quad (3.15)$$

their wave functions being

$$\langle \mathbf{r}_e, \mathbf{r}_{e'}, \mathbf{r}_h|\eta, \mathbf{K}\rangle = \langle \mathbf{r}, \mathbf{u}|\eta\rangle\langle \mathbf{R}_T|\mathbf{K}\rangle. \quad (3.16)$$

As H_T, given in equation (3.12), stays invariant under the $(e \leftrightarrow e')$ exchange, its eigenstates are odd or even with respect to this exchange. Since $\mathbf{R}_T$ is unchanged, this means that the $\langle \mathbf{r}, \mathbf{u}|\eta\rangle$'s are odd or even. Let us call η_0 (resp. η_1), the η indices which correspond to even (resp. odd) functions with respect to $(e \leftrightarrow e')$ exchange. Due to equation (3.11), this parity condition reads, for $S = (0, 1)$,

$$\begin{aligned}\langle \mathbf{r}, \mathbf{u}|\eta_S\rangle &= (-1)^S\langle \mathbf{r}', \mathbf{u}'|\eta_S\rangle \\ &= (-1)^S\langle \mathbf{u} + \alpha_e\mathbf{r}, (1-\alpha_e^2)\mathbf{r} - \alpha_e\mathbf{u}|\eta_S\rangle, \quad (3.17)\end{aligned}$$

which is definitely not very appealing. It is actually possible to rewrite this condition in a nicer form for physical understanding, by using other variables than $(\mathbf{r}, \mathbf{u})$, namely $(\nu, \mathbf{p})$:

$$\langle \nu, \mathbf{p}|\eta_S\rangle = \int d\mathbf{r}\, d\mathbf{u}\, \langle \nu|\mathbf{r}\rangle\langle \mathbf{p}|\mathbf{u}\rangle\langle \mathbf{r}, \mathbf{u}|\eta_S\rangle. \quad (3.18)$$

Equation (3.18) corresponds to a Fourier transform "in the exciton sense". If we insert equation (3.17) into equation (3.18) and replace $\langle \mathbf{r}', \mathbf{u}'|\eta_S\rangle$ by $\sum_{\nu',\mathbf{p}'}\langle \mathbf{r}'|\nu'\rangle\langle \mathbf{u}'|\mathbf{p}'\rangle\langle \nu', \mathbf{p}'|\eta_S\rangle$, we find, by expressing all spatial variables in terms of $\mathbf{r}$ and $\mathbf{r}'$ (through $\mathbf{u} = \mathbf{r}' - \alpha_e\mathbf{r}$ and $\mathbf{u}' = \mathbf{r} - \alpha_e\mathbf{r}'$) and using $\langle \mathbf{p}|\mathbf{r}' - \alpha_e\mathbf{r}\rangle\langle \mathbf{r} - \alpha_e\mathbf{r}'|\mathbf{p}'\rangle \equiv \langle \mathbf{p} + \alpha_e\mathbf{p}'|\mathbf{r}'\rangle\langle \mathbf{r}|p' + \alpha_e\mathbf{p}\rangle$, that the $|\eta_S\rangle$'s fulfilling equation (3.17) are such that

$$\langle \nu, \mathbf{p}|\eta_S\rangle = (-1)^S \sum_{\nu',\mathbf{p}'} L_{\nu\mathbf{p};\nu'\mathbf{p}'}\, \langle \nu', \mathbf{p}'|\eta_S\rangle, \quad (3.19)$$

where $L_{\nu\mathbf{p};\nu'\mathbf{p}'}$ appears as

$$L_{\nu\mathbf{p};\nu'\mathbf{p}'} = \langle \nu|\mathbf{p}' + \alpha_e\mathbf{p}\rangle\, \langle \mathbf{p} + \alpha_e\mathbf{p}'|\nu'\rangle. \quad (3.20)$$

As shown below, this $L_{\nu\mathbf{p};\nu'\mathbf{p}'}$ is a crucial parameter: It is just the exchange "scattering" of the "commutation technique" for excitons interacting with electrons. Its link with electron exchange inside an e-X pair can however be made apparent right now, by noting that

$$\int d\mathbf{r}_e\, d\mathbf{r}_{e'}\, d\mathbf{r}_h\, \phi^*_{\nu',\mathbf{Q}'}(\mathbf{r}_{e'}, \mathbf{r}_h)\, f^*_{\mathbf{k}'}(\mathbf{r}_e)\, \phi_{\nu,\mathbf{Q}}(\mathbf{r}_e, \mathbf{r}_h)\, f_{\mathbf{k}}(\mathbf{r}_{e'}) = \delta_{\mathbf{K}',\mathbf{K}}\, L_{\nu'\mathbf{p}';\nu\mathbf{p}}, \quad (3.21)$$

where $\phi_{\nu,\mathbf{Q}}(\mathbf{r}_e, \mathbf{r}_h) = \langle \mathbf{r}_e, \mathbf{r}_h|\nu, \mathbf{Q}\rangle$ is the exciton wave function, $f_{\mathbf{k}}(\mathbf{r}) = \langle \mathbf{r}|\mathbf{k}\rangle$ the free electron wave function while the $(\mathbf{Q}, \mathbf{k})$ and $(\mathbf{K}, \mathbf{p})$ are linked by

$$\mathbf{K} = \mathbf{Q} + \mathbf{k}, \qquad \mathbf{k} = \mathbf{p} + \beta_e\mathbf{K}$$
$$\beta_e = 1 - \beta_X = \frac{m_e}{M_T}\cdot \quad (3.22)$$

In equation (3.21), the "in" exciton $(\nu, \mathbf{Q})$ and the "out" exciton $(\nu', \mathbf{Q}')$ are made with *different* electrons, $\mathbf{r}_e$ and $\mathbf{r}_{e'}$, the corresponding process being shown in Figure 5a. As two exchanges reduce to an identity, we do have

$$\sum_{\nu'',\mathbf{p}''} L_{\nu'\mathbf{p}';\nu''\mathbf{p}''}\, L_{\nu''\mathbf{p}'';\nu\mathbf{p}} = \delta_{\nu',\nu}\, \delta_{\mathbf{p}',\mathbf{p}}, \quad (3.23)$$

easy to check from equation (3.20).

If we now come back to the trion relative motion eigenstates, the η index is actually an η_0 if the trion state has a total electron spin $S = 0$ and an η_1 if its total spin is $S = 1$:

$$\eta = \eta_0\, \delta_{S,0} + \eta_1\, \delta_{S,1}, \quad (3.24)$$

where S is the trion electronic spin. The trion ground state having a symmetrical orbital wave function, its index belongs to the η_0 set, its total electronic spin being $S = 0$. These $|\eta\rangle$'s form an orthogonal basis, $\langle \eta'|\eta\rangle = \delta_{\eta',\eta}$, their closure relation reading $\sum_\eta |\eta\rangle\langle\eta| = I$, with the sum taken over the η_0's and η_1's, so that $I = I_0 + I_1$, with $I_S = \sum_{\eta_S} |\eta_S\rangle\langle\eta_S|$. An interesting relation also exists for the partial sum I_S:

$$\langle \nu', \mathbf{p}'|I_S|\nu, \mathbf{p}\rangle = \frac{1}{2}\left(\delta_{\nu',\nu}\, \delta_{\mathbf{p}',\mathbf{p}} + (-1)^S L_{\nu'\mathbf{p}';\nu\mathbf{p}}\right), \quad (3.25)$$

as can be shown by noting that, due to equation (3.19), $\langle\nu',\mathbf{p}'|I_S|\nu,\mathbf{p}\rangle = (-1)^S \sum_{\nu'',\mathbf{p}''} L_{\nu'\mathbf{p}';\nu''\mathbf{p}''}$ $\langle\nu'',\mathbf{p}''|I_S|\nu,\mathbf{p}\rangle$, so that, while $\langle\nu',\mathbf{p}'|I_0 + I_1|\nu,\mathbf{p}\rangle = \delta_{\nu',\nu}\,\delta_{\mathbf{p}',\mathbf{p}}$, we do have $\langle\nu',\mathbf{p}'|I_0 - I_1|\nu,\mathbf{p}\rangle = L_{\nu'\mathbf{p}';\nu\mathbf{p}}$. Equation (3.25) follows from the combination of these two results.

Finally, the closure relation for the free states $|\nu,\mathbf{p}\rangle$ leads to write $|\eta\rangle$ as $\sum_{\nu,\mathbf{p}} |\nu,\mathbf{p}\rangle\langle\nu,\mathbf{p}|\eta\rangle$. So that the projection over $\langle\nu,\mathbf{p}|$ of the Schrödinger equation for $|\eta\rangle$ gives

$$\left(\varepsilon_\nu^{(X)} + \epsilon_{\mathbf{p}}^{(eX)} - \varepsilon_\eta^{(T)}\right)\langle\nu,\mathbf{p}|\eta\rangle + \sum_{\nu',\mathbf{p}'}\langle\nu,\mathbf{p}|w(\mathbf{r},\mathbf{u})|\nu',\mathbf{p}'\rangle\,\langle\nu',\mathbf{p}'|\eta\rangle = 0, \quad (3.26)$$

the scattering being linked to the Fourier transform "in the exciton sense" of the Coulomb potential $w(\mathbf{r},\mathbf{u})$ appearing in h_T,

$$\langle\nu,\mathbf{p}|w(\mathbf{r},\mathbf{u})|\nu',\mathbf{p}'\rangle = W^{\nu\nu'}_{\mathbf{p}'-\mathbf{p}} \equiv \langle\nu|w_{\mathbf{p}-\mathbf{p}'}(\mathbf{r})|\nu'\rangle, \quad (3.27)$$

where $w_{\mathbf{q}}(\mathbf{r}) = V_{\mathbf{q}}(e^{-i\alpha_h\mathbf{q}.\mathbf{r}} - e^{i\alpha_e\mathbf{q}.\mathbf{r}})$ is the Fourier transform of $w(\mathbf{r},\mathbf{u})$ with respect to the variable $\mathbf{u}$. Note that these results for trions are formally similar to the ones for excitons, given in equations (3.4–5), except for the additional exciton quantum number ν. It turns out that this scattering is just the direct Coulomb scattering $C^{\rm dir}_{\nu\mathbf{p};\nu'\mathbf{p}'}$ of the "commutation technique" for excitons interacting with electrons, which will be introduced below:

$$W^{\nu\nu'}_{\mathbf{p}'-\mathbf{p}} \equiv C^{\rm dir}_{\nu\mathbf{p};\nu'\mathbf{p}'}. \quad (3.28)$$

Its link to direct Coulomb processes is however easy to see right now, since

$$\int d\mathbf{r}_e\, d\mathbf{r}_{e'}\, d\mathbf{r}_h\, \phi^*_{\nu',\mathbf{Q}'}(\mathbf{r}_e,\mathbf{r}_h)\, f^*_{\mathbf{k}'}(\mathbf{r}_{e'}) \times\left(\frac{e^2}{|\mathbf{r}_{e'}-\mathbf{r}_e|} - \frac{e^2}{|\mathbf{r}_{e'}-\mathbf{r}_h|}\right)\phi_{\nu,\mathbf{Q}}(\mathbf{r}_e,\mathbf{r}_h)\, f_{\mathbf{k}}(\mathbf{r}_{e'}) = \delta_{\mathbf{K}',\mathbf{K}}\, C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}}, \quad (3.29)$$

where $(\mathbf{Q},\mathbf{k};\mathbf{K},\mathbf{p})$ and $(\mathbf{Q}',\mathbf{k}';\mathbf{K}',\mathbf{p}')$ are linked by equation (3.22), the "in" exciton $(\nu,\mathbf{Q})$ and the "out" exciton $(\nu',\mathbf{Q}')$ being here made with the *same* electron $(\mathbf{r}_e)$, see Figure 5b.

3.3 Trions in second quantization

3.3.1 Creation operators of e-X pairs

If we look at the expression of the exciton creation operator in terms of e-h pairs, equation (3.7), we see that the exciton center of mass momentum $\mathbf{Q}$ is split between (e, h) according to their masses, namely $\alpha_e\mathbf{Q}_p$ and $\alpha_h\mathbf{Q}_p$. In a similar way, we are led to split the center of mass momentum $\mathbf{K}$ of the e-X pair between (e,X) according to their masses, namely

$$\mathcal{T}^\dagger_{\nu,\mathbf{p},\mathbf{K};\sigma,s,m} = a^\dagger_{\mathbf{p}+\beta_e\mathbf{K},\sigma}\, B^\dagger_{\nu,-\mathbf{p}+\beta_X\mathbf{K},s,m}, \quad (3.30)$$

with β_e, β_X given in equation (3.22).

In order to calculate the scalar product of these e-X states, it is convenient to introduce the "commutation technique" for excitons interacting with electrons. From the deviation-from-boson operator $D_{n'n}$ defined as [22,23]

$$[B_{n'}, B^\dagger_n] = \delta_{n',n} - D_{n'n}, \quad (3.31)$$

where the $B^\dagger_n$'s are the exciton creation operators, equation (3.7), and n stands for $(\nu,\mathbf{Q},s,m)$ while n' stands for $(\nu',\mathbf{Q}',s',m')$, we find

$$[D_{n'n}, a^\dagger_{\mathbf{k},\sigma}] = \delta_{m',m}\,\delta_{s',\sigma}\,\delta_{\mathbf{K}',\mathbf{K}}\sum_{\mathbf{k}'} L_{\nu'\mathbf{p}';\nu\mathbf{p}}\, a^\dagger_{\mathbf{k}',s}, \quad (3.32)$$

with $(\mathbf{Q},\mathbf{k};\mathbf{K},\mathbf{p})$ and $(\mathbf{Q}',\mathbf{k}';\mathbf{K}',\mathbf{p}')$ again linked by equation (3.22), $L_{\nu'\mathbf{p}';\nu\mathbf{p}}$ being the parameter already appearing in equations (3.19−21) (see Fig. 5a).

From equations (3.31−32), we then readily find

$$\langle v|\mathcal{T}_{\nu',\mathbf{p}',\mathbf{K}';\sigma',s',m'}\mathcal{T}^\dagger_{\nu,\mathbf{p},\mathbf{K};\sigma,s,m}|v\rangle = \delta_{m',m}\,\delta_{\mathbf{K}',\mathbf{K}}\,(\delta_{\sigma',\sigma}\,\delta_{s',s}\,\delta_{\nu',\nu}\,\delta_{\mathbf{p}',\mathbf{p}} - \delta_{\sigma',s}\,\delta_{s',\sigma}\,L_{\nu'\mathbf{p}';\nu\mathbf{p}}). \quad (3.33)$$

It will also be useful to note that

$$\mathcal{T}^\dagger_{\nu,\mathbf{p},\mathbf{K};\sigma,s,m} = -\sum_{\nu',\mathbf{p}'} L_{\nu'\mathbf{p}';\nu\mathbf{p}}\,\mathcal{T}^\dagger_{\nu',\mathbf{p}',\mathbf{K};s,\sigma,m}, \quad (3.34)$$

which results from the two possible ways to construct an e-X pair out of e-e-h. (Note that the electron spins are exchanged from the right to the left of Eq. (3.34).)

In the same way as exciton reads in terms of e-h pairs, trion reads in terms of e-X pairs. The simplest way to get this decomposition is to first find how the semiconductor Hamiltonian H acts on one e-X pair. For that, we again use the "commutation technique" for excitons interacting with electrons. From the Coulomb creation operator $V^\dagger_n$ defined as [22,23]

$$[H, B^\dagger_n] = E^{(X)}_n\, B^\dagger_n + V^\dagger_n, \quad (3.35)$$

we find

$$[V^\dagger_n, a^\dagger_{\mathbf{k},\sigma}] = \sum_{\nu',\mathbf{p}'} C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}}\,\mathcal{T}^\dagger_{\nu',\mathbf{p}',\mathbf{K};\sigma,s,m}, \quad (3.36)$$

where $(\mathbf{Q},\mathbf{k};\mathbf{K},\mathbf{p})$ are linked by equation (3.22), $C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}}$ being the quantity already appearing in equations (3.28−29). From equations (3.30, 35, 36), we readily get

$$H\,\mathcal{T}^\dagger_{\nu,\mathbf{p},\mathbf{K};\sigma,s,m}|v\rangle = E_{\nu\mathbf{p}\mathbf{K}}\,\mathcal{T}^\dagger_{\nu,\mathbf{p},\mathbf{K};\sigma,s,m}|v\rangle + \sum_{\nu',\mathbf{p}'} C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}}\,\mathcal{T}^\dagger_{\nu',\mathbf{p}',\mathbf{K};\sigma,s,m}|v\rangle, \quad (3.37)$$

where $E_{\nu\mathbf{pK}}$ is the energy of the free e-X pair $(\nu, \mathbf{p}, \mathbf{K})$,

$$E_{\nu\mathbf{pK}} = \varepsilon_{\nu}^{(X)} + \epsilon_{\mathbf{p}}^{(eX)} + \mathcal{E}_{\mathbf{K}}^{(T)}, \tag{3.38}$$

with $\epsilon_{\mathbf{p}}^{(eX)}$ and $\mathcal{E}_{\mathbf{K}}^{(T)}$ being the relative motion and center of mass energies of the e-X pair, defined in equations (3.14−15). Note that, for $(\mathbf{Q}, \mathbf{k}; \mathbf{K}, \mathbf{p})$ linked by equation (3.22), we do have

$$\epsilon_{\mathbf{k}}^{(e)} + \mathcal{E}_{\mathbf{Q}}^{(X)} = \epsilon_{\mathbf{p}}^{(eX)} + \mathcal{E}_{\mathbf{K}}^{(T)}. \tag{3.39}$$

3.3.2 Creation operators for ($S_z = \pm 1$) trions

Trions with $S_z = \pm 1$ have a total spin $S = 1$, so that their relative motion index belongs to the η_1 set. Moreover, they are constructed from a $\sigma = \pm 1/2$ electron and an $s = \pm 1/2$ exciton. Let us introduce [18]

$$\mathrm{T}^{\dagger}_{\eta_1,\mathbf{K};1,\pm1,m} = \frac{1}{\sqrt{2}} \sum_{\nu,\mathbf{p}} \langle \nu, \mathbf{p} | \eta_1 \rangle \, \mathcal{T}^{\dagger}_{\nu,\mathbf{p},\mathbf{K};\pm1/2,\pm1/2,m}, \tag{3.40}$$

which is similar to the exciton creation operator, equation (3.7), except for the $1/\sqrt{2}$ prefactor which is made for this operator to create a normalized e-e-h state; indeed, from equations (3.19, 33),

$$\langle v | \mathrm{T}_{\eta_1,\mathbf{K};1,\pm1,m} \, \mathrm{T}^{\dagger}_{\eta_1,\mathbf{K};1,\pm1,m} | v \rangle = \frac{1}{2} \, 2 \sum_{\nu,\mathbf{p}} |\langle \nu, \mathbf{p} | \eta_1 \rangle|^2 = 1. \tag{3.41}$$

This $\mathrm{T}^{\dagger}_{\nu,\mathbf{K};1,\pm1,m}$ is indeed the creation operator for $(S = 1, S_z = \pm 1)$ trions, since, due to equations (3.26, 37), we do have

$$H \, \mathrm{T}^{\dagger}_{\eta_1,\mathbf{K};1,\pm1,m} | v \rangle = E^{(T)}_{\eta_1,\mathbf{K}} \, \mathrm{T}^{\dagger}_{\eta_1,\mathbf{K};1,\pm1,m} | v \rangle, \tag{3.42}$$

with

$$E^{(T)}_{\eta_S,\mathbf{K}} = \varepsilon^{(T)}_{\eta_S} + \mathcal{E}^{(T)}_{\mathbf{K}}. \tag{3.43}$$

3.3.3 Creation operators for ($S_z = 0$) trions

($S_z = 0$) trions have a total spin $S = 0$ or $S = 1$, so that their relative motion indices can be either an η_0 or an η_1. Moreover, they can be built either from a $(\sigma = 1/2)$ electron and a $(s = -1/2)$ exciton, or the reverse. However, as the two corresponding e-X operators are linked by equation (3.34), we can just use one type of e-X pairs [18],

$$\mathrm{T}^{\dagger}_{\eta_S,\mathbf{K};S,0,m} = \sum_{\nu,\mathbf{p}} \langle \nu, \mathbf{p} | \eta_S \rangle \, \mathcal{T}^{\dagger}_{\nu,\mathbf{p},\mathbf{K};+1/2,-1/2,m}, \tag{3.44}$$

as, from equations (3.19, 34), this operator also reads

$$\mathrm{T}^{\dagger}_{\eta_S,\mathbf{K};S,0,m} = -(-1)^S \sum_{\nu,\mathbf{p}} \langle \nu, \mathbf{p} | \eta_S \rangle \, \mathcal{T}^{\dagger}_{\nu,\mathbf{p},\mathbf{K};-1/2,+1/2,m}. \tag{3.45}$$

Using again equations (3.19, 33) and (3.26, 37), it is straightforward to check that this $\mathrm{T}^{\dagger}_{\eta_S,\mathbf{K};S,0,m}$ is indeed a normalized trion creation operator:

$$H \, \mathrm{T}^{\dagger}_{\eta_S,\mathbf{K};S,0,m} | v \rangle = E^{(T)}_{\eta_S,\mathbf{K}} \, \mathrm{T}^{\dagger}_{\eta_S,\mathbf{K};S,0,m} | v \rangle. \tag{3.46}$$

Let us end by noting that these trion states form an orthogonal basis,

$$\langle v | \mathrm{T}_{\eta'_{S'},\mathbf{K}';S',S'_z,m'} \, \mathrm{T}^{\dagger}_{\eta_S,\mathbf{K};S,S_z,m} | v \rangle = \delta_{S',S} \, \delta_{S'_z,S_z} \, \delta_{m',m} \, \delta_{\eta'_{S'},\eta_S} \, \delta_{\mathbf{K}',\mathbf{K}}, \tag{3.47}$$

while their closure relation reads

$$1 = \sum_{S,S_z,m,\eta_S,\mathbf{K}} \mathrm{T}^{\dagger}_{\eta_S,\mathbf{K};S,S_z,m} | v \rangle \, \langle v | \mathrm{T}_{\eta_S,\mathbf{K};S,S_z,m}. \tag{3.48}$$

Finally, it is easy to check that, in the same way as trions can be written in terms of e-X pairs according to equations (3.40, 44), e-X pairs can be written in terms of trions, according to

$$\mathcal{T}^{\dagger}_{\nu,\mathbf{p},\mathbf{K};\pm1/2;\pm1/2,m} = \sqrt{2} \sum_{\eta_1} \langle \eta_1 | \nu, \mathbf{p} \rangle \, \mathrm{T}^{\dagger}_{\eta_1,\mathbf{K};1,\pm1,m}, \tag{3.49}$$

$$\mathcal{T}^{\dagger}_{\nu,\mathbf{p},\mathbf{K};+1/2;-1/2,m} = \sum_{S,\eta_S} \langle \eta_S | \nu, \mathbf{p} \rangle \mathrm{T}^{\dagger}_{\eta_S,\mathbf{K};S,0,m}, \tag{3.50}$$

$$\mathcal{T}^{\dagger}_{\nu,\mathbf{p},\mathbf{K};-1/2;+1/2,m} = -\sum_{S,\eta_S} (-1)^S \langle \eta_S | \nu, \mathbf{p} \rangle \, \mathrm{T}^{\dagger}_{\eta_S,\mathbf{K};S,0,m}. \tag{3.51}$$

Although somewhat more complicated due to spins, these equations are the analogues of equation (3.8) for free e-h pairs in terms of excitons.

3.4 Many-body effects between electrons and excitons

In usual many-body problems, the Hamiltonian splits as $H = H_0 + V$, so that the many-body effects result from equation (2.5) and its iteration (2.6). In the case of many-body effects with excitons, such a separation of the Hamiltonian is not possible, due to the composite nature of the exciton. Attempts have been made to produce a potential V_{XX} between excitons by bosonizing them. However, we have quite recently shown that these procedures, although rather sophisticated, fail to give the correct answer to physical quantities such as the exciton lifetime and scattering rates [25], whatever the X-X scattering used in V_{XX} is. We have also shown that they fail to give correct nonlinear susceptibilities [26], because they miss purely Pauli many-body effects, i.e., scatterings in the absence of Coulomb interaction. This is why we are not going here to bosonize the excitons.

It is actually possible to handle many-body effects with excitons properly, by noting that equation (3.35) leads to [24]

$$\frac{1}{a - H} \, B^{\dagger}_n = B^{\dagger}_n \frac{1}{a - H - E^{(X)}_n} + \frac{1}{a - H} \, V^{\dagger}_n \frac{1}{a - H - E^{(X)}_n}. \tag{3.52}$$

$$A_{\nu''\mathbf{p}'';\nu\mathbf{p}}(a,\mathbf{K}) = \sum_{S',S'_Z,m',\eta'_{S'},\mathbf{K}'} \frac{\langle v|\mathcal{T}_{\nu'',\mathbf{p}'',\mathbf{K};+1/2,-1/2,m}\,\mathrm{T}^{\dagger}_{\eta'_{S'},\mathbf{K}';S',S'_z;m'}|v\rangle\langle v|\mathrm{T}_{\eta'_{S'},\mathbf{K}';S',S'_z;m'}\,\mathcal{T}^{\dagger}_{\nu,\mathbf{p},\mathbf{K};+1/2,-1/2,m}|v\rangle}{a - E^{(T)}_{\eta'_{S'},\mathbf{K}'}}. \quad (3.57)$$

The above equation, which is the equivalent of equation (2.5) for usual many-body effects, is the key equation for many-body effects with excitons. While it cannot be iterated as simply as equation (2.6), a similar iteration can be generated by having equation (3.52) acting on excitons or on electrons and by using either $[V_n^{\dagger}, B_n^{\dagger}]$ given in equation (3) of reference [22] for many-body effects between excitons, or $[V_n^{\dagger}, a^{\dagger}_{\mathbf{k},\sigma}]$, given in equation (3.36), for many-body effects between excitons and electrons.

3.4.1 $(a - H)^{-1}$ acting on e-X pairs

Equations (3.36, 52) give $(a-H)^{-1}$ acting on one e-X pair as [18]

$$\frac{1}{a-H}\,\mathcal{T}^{\dagger}_{\nu,\mathbf{p},\mathbf{K};\sigma,s,m}|v\rangle = \left[\mathcal{T}^{\dagger}_{\nu,\mathbf{p},\mathbf{K};\sigma,s,m}|v\rangle + \sum_{\nu',\mathbf{p}'} \frac{1}{a-H}\,\mathcal{T}^{\dagger}_{\nu',\mathbf{p}',\mathbf{K};\sigma,s,m}|v\rangle C^{\mathrm{dir}}_{\nu'\mathbf{p}';\nu\mathbf{p}}\right] \times \frac{1}{a - E_{\nu\mathbf{p}\mathbf{K}}}, \quad (3.53)$$

where $E_{\nu\mathbf{p}\mathbf{K}}$ is the free e-X pair energy, equation (3.38).

If we now iterate equation (3.53), we find

$$\frac{1}{a-H}\,\mathcal{T}^{\dagger}_{\nu,\mathbf{p},\mathbf{K};\sigma,s,m}|v\rangle = \sum_{\nu',\mathbf{p}'} A_{\nu'\mathbf{p}';\nu\mathbf{p}}(a,\mathbf{K})\,\mathcal{T}^{\dagger}_{\nu',\mathbf{p}',\mathbf{K};\sigma,s,m}|v\rangle, \quad (3.54)$$

where $A_{\nu'\mathbf{p}';\nu\mathbf{p}}(a,\mathbf{K})$ expands on the e-X *direct* Coulomb scatterings as

$$A_{\nu'\mathbf{p}';\nu\mathbf{p}}(a,\mathbf{K}) = \frac{\delta_{\nu',\nu}\,\delta_{\mathbf{p}',\mathbf{p}}}{a - E_{\nu\mathbf{p}\mathbf{K}}} + \frac{C^{\mathrm{dir}}_{\nu'\mathbf{p}';\nu\mathbf{p}}}{(a - E_{\nu'\mathbf{p}'\mathbf{K}})(a - E_{\nu\mathbf{p}\mathbf{K}})} + \sum_{\nu_1,\mathbf{p}_1} \frac{C^{\mathrm{dir}}_{\nu'\mathbf{p}';\nu_1\mathbf{p}_1} C^{\mathrm{dir}}_{\nu_1\mathbf{p}_1;\nu\mathbf{p}}}{(a - E_{\nu'\mathbf{p}'\mathbf{K}})(a - E_{\nu_1\mathbf{p}_1\mathbf{K}})(a - E_{\nu\mathbf{p}\mathbf{K}})} + \cdots \quad (3.55)$$

It corresponds to the ladder processes between electron and exciton shown in Figure 6.

Just as the summation of the e-h ladder processes producing the exciton reads in terms of exciton energies and wave functions, the summation of these e-X ladder processes producing the trion should read in terms of trion energies and wave functions. We now show it.

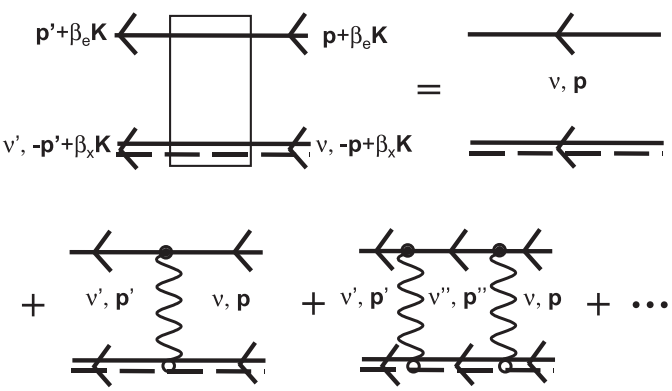

Fig. 6. Set of e-X ladder processes. All over, the exciton is made with the same electron and the e-X pair has the same center of mass momentum $\mathbf{K}$. The exciton quantum number, ν, and the pair relative motion momentum, $\mathbf{p}$, are the only things which change in these ladder processes. The solid line corresponds to the electron. The double solid-dashed line corresponds to the exciton. The wavy line corresponds to the e-X direct Coulomb scattering $C^{\mathrm{dir}}_{\nu'\mathbf{p}';\nu\mathbf{p}}$.

3.4.2 Sum of e-X ladder processes

If we take the scalar product of $\langle v|\mathcal{T}_{\nu'',\mathbf{p}'',\mathbf{K};+1/2,-1/2,m}$ with equation (3.54) for $\sigma = -s = 1/2$, we find from equation (3.33)

$$\langle v|\mathcal{T}_{\nu'',\mathbf{p}'',\mathbf{K};+1/2,-1/2,m}\frac{1}{a-H}\,\mathcal{T}^{\dagger}_{\nu,\mathbf{p},\mathbf{K};+1/2,-1/2,m}|v\rangle = A_{\nu''\mathbf{p}'';\nu\mathbf{p}}(a,\mathbf{K}). \quad (3.56)$$

If we now insert the trion closure relation (3.48) in the above LHS, we get, using equations (3.42, 46),

see equation (3.57) above.

From the expansion of e-X pairs in terms of trions, equation (3.50), we immediately find, since trions form an orthogonal basis,

$$A_{\nu''\mathbf{p}'';\nu\mathbf{p}}(a,\mathbf{K}) = \sum_{S=0,1} A^{(S)}_{\nu''\mathbf{p}'';\nu\mathbf{p}}(a,\mathbf{K}), \quad (3.58)$$

$$A^{(S)}_{\nu''\mathbf{p}'';\nu\mathbf{p}}(a,\mathbf{K}) = \sum_{\eta_S} \frac{\langle\nu'',\mathbf{p}''|\eta_S\rangle\langle\eta_S|\nu,\mathbf{p}\rangle}{a - E^{(T)}_{\eta_S,\mathbf{K}}}. \quad (3.59)$$

$A_{\nu''\mathbf{p}'';\nu\mathbf{p}}(a,\mathbf{K})$ contains contributions from both, singlet and triplet trions ($S = 0, 1$).

We can note that, if $a' = a - \mathcal{E}_{\mathbf{K}}^{(T)}$, the expression of $A_{\nu''\mathbf{p}'';\nu\mathbf{p}}(a,\mathbf{K})$ in terms of trions, given in equations (3.58–59), reads

$$A_{\nu''\mathbf{p}'';\nu\mathbf{p}}(a,\mathbf{K}) = \hat{A}_{\nu''\mathbf{p}'';\nu\mathbf{p}}(a') = \langle\nu'',\mathbf{p}''|\frac{1}{a'-h_{\mathrm{T}}}|\nu,\mathbf{p}\rangle, \tag{3.60}$$

while its expansion in e-X Coulomb scatterings given in equation (3.55) just corresponds to

$$\hat{A}_{\nu''\mathbf{p}'';\nu\mathbf{p}}(a') = \langle\nu'',\mathbf{p}''|\frac{1}{a'-h_T^{(0)}}|\nu,\mathbf{p}\rangle + \langle\nu'',\mathbf{p}''|\frac{1}{a'-h_T^{(0)}}w(\mathbf{r},\mathbf{u})\frac{1}{a'-h_T^{(0)}}|\nu,\mathbf{p}\rangle + \cdots, \tag{3.61}$$

due to the link between $C^{\mathrm{dir}}_{\nu'\mathbf{p}';\nu\mathbf{p}}$ and $w(\mathbf{r},\mathbf{u})$ given in equations (3.27–28). It is then obvious that equation (3.61) just follows from equation (3.60), since for $h_T = h_T^{(0)} + w(\mathbf{r},\mathbf{u})$, we do have

$$\frac{1}{a'-h_T} = \frac{1}{a'-h_T^{(0)}} + \frac{1}{a'-h_T^{(0)}}\,w(\mathbf{r},\mathbf{u})\,\frac{1}{a'-h_T^{(0)}} + \cdots \tag{3.62}$$

Let us stress that this summation of e-X ladder processes has been established from a quite formal procedure designed to treat e-X interactions. It can a priori be used in *any* other problem dealing with electrons and excitons, not just in the simplest one: one e-X pair, i.e., one trion. It is however clear that it can also be used in this simple problem too, for which a first quantization formulation, through the trion Hamiltonian $H_T(\mathbf{R}_T,\mathbf{r},\mathbf{u})$, is simple enough to be of practical use. This first quantization formulation of the one-trion problem actually provides enlightening foreshortenings to some results on e-X pairs. However, second quantization along with the formal definitions of direct Coulomb and exchange scatterings between electrons and excitons, are the only possible way to treat more complicated problems with one hole plus many electrons. The two aspects of the same results are however of interest for the understanding of the trion physics.

4 Photon absorption using electron-exciton diagrams

In Section 2, we have derived the photon absorption using e-h diagrams in the cases of exciton and trion formations. We have shown that these standard Feynman diagrams are totally inappropriate for trions. In Section 3, we have derived all the tools necessary to propose a new diagrammatic procedure for photon absorption in terms of excitons, while taking into account the fact that the exciton can be made with any of the electrons present in the sample, through exchange "scatterings", as generated by the "commutation technique".

This new formulation of photon absorption is in fact quite natural: Indeed, the semiconductor-photon interaction, given in equation (2.3) in terms of free electrons and holes, can also be written in terms of excitons. From equation (3.8) and the fact that $\sqrt{\Omega}\langle\mathbf{p}|\mathbf{r}=\mathbf{0}\rangle = 1$, we readily get

$$U_{\pm}^{\dagger}(\mathbf{Q}_p) = \sum_{\nu}\lambda_{\nu}^{*}\,B^{\dagger}_{\nu,\mathbf{Q}_p,\mp,\pm}, \tag{4.1}$$

$$\lambda_{\nu}^{*} = \lambda^{*}\sqrt{\Omega}\langle\nu|\mathbf{r}=\mathbf{0}\rangle. \tag{4.2}$$

This just corresponds to the well known enhancement factor of the coupling to exciton instead of free pairs, as $|\langle\nu|\mathbf{r}=\mathbf{0}\rangle|^2$ is of the order of the inverse exciton volume: Just from this enhanced coupling, the exciton representation can already appear somewhat better.

4.1 Photon absorption with exciton formation

The response function, equation (2.4), for a semiconductor with no carrier, irradiated by σ_+ photons, also reads

$$S_X = N_p\sum_{\nu',\nu}\lambda_{\nu'}\langle v|B_{\nu',\mathbf{Q},-,+}\,\frac{1}{\omega_p - H + i\eta}\,B^{\dagger}_{\nu,\mathbf{Q}_p,-,+}|v\rangle\,\lambda_{\nu}^{*}. \tag{4.3}$$

$$= N_p\sum_{\nu}\lambda_{\nu}\,G_X(\omega_p,\mathbf{Q}_p;\nu)\,\lambda_{\nu}^{*}, \tag{4.4}$$

where $G_X(\omega,\mathbf{Q};\nu)$, defined as

$$G_X(\omega,\mathbf{Q};\nu) = \frac{1}{\omega - E^{(X)}_{\nu,\mathbf{Q}} + i\eta}, \tag{4.5}$$

can be seen as an exciton propagator. This leads to represent S_X by the exciton diagram of Figure 7a, which is already far simpler than the set of e-h ladder diagrams of Figure 1.

4.2 Photon absorption with trion formation

From equations (2.4,4.1), the response function for an initial state with one $\mathbf{k}_i, s_i$ electron and an absorbed photon $\sigma_{\pm}$, reads

$$S_{\pm,s_i} = N_p\sum_{\nu',\nu}\lambda_{\nu'}\,\langle v|a_{\mathbf{k}_i,s_i}\,B_{\nu',\mathbf{Q}_p,\mp,\pm} \times\left(\frac{1}{\omega_p + \epsilon^{(e)}_{\mathbf{k}_i} - H + i\eta}\right)B^{\dagger}_{\nu,\mathbf{Q}_p,\mp,\pm}\,a^{\dagger}_{\mathbf{k}_i,s_i}|v\rangle\,\lambda_{\nu}^{*}. \tag{4.6}$$

If we introduce the appropriate momenta $(\mathbf{p}_i,\mathbf{K}_i)$ of the e-X pair made of the initial electron and the photocreated virtual exciton, defined by

$$\mathbf{K}_i = \mathbf{k}_i + \mathbf{Q}_p, \quad \mathbf{k}_i = \mathbf{p}_i + \beta_e\mathbf{K}_i, \tag{4.7}$$

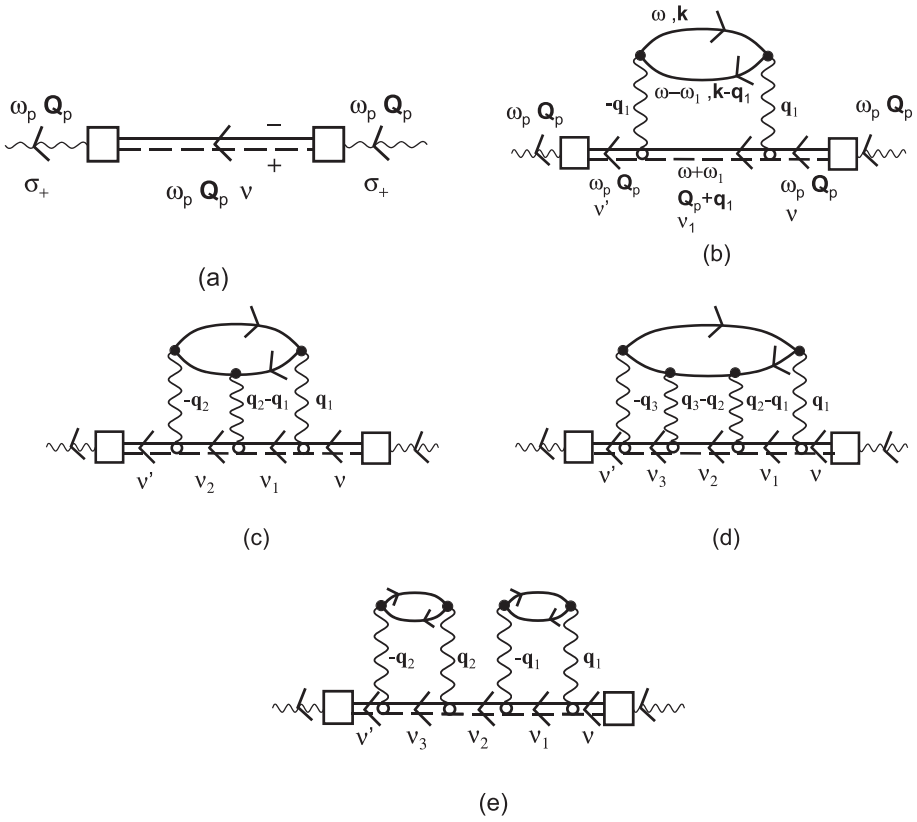

Fig. 7. (a) Absorption of a σ_+ photon with exciton formation, using exciton diagrams. This unique diagram corresponds to the set of e-h ladder diagrams of Figure 1. It also corresponds to the trion absorption diagram, at zero order in Coulomb scattering $C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}}$. (b) Absorption of a σ_+ photon with trion formation, when the photocreated and initial electrons have different spins: second order process in e-X (direct) Coulomb scattering, $C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}}$. (c) and (d) Same as (b), with three and four direct Coulomb scatterings. (e) Additional diagram which appears in the response function at fourth order in $C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}}$.

we can rewrite $S_{\pm,s_i}$ as

$$S_{\pm,s_i} = N_p \sum_{\nu',\nu} \lambda_{\nu'} \langle v|\mathcal{T}_{\nu',\mathbf{p}_i,\mathbf{K}_i;s_i,\mp,\pm} \times \left(\frac{1}{\omega_p + \epsilon^{(e)}_{\mathbf{k}_i} - H + i\eta}\right) \mathcal{T}^\dagger_{\nu,\mathbf{p}_i,\mathbf{K}_i;s_i,\mp,\pm}|v\rangle\, \lambda^*_\nu \quad (4.8)$$

$$= N_p \sum_{\nu',\nu} \sum_{\nu_1,\mathbf{p}_1} \lambda_{\nu'} \langle v|\mathcal{T}_{\nu',\mathbf{p}_i,\mathbf{K}_i;s_i,\mp,\pm}\, \mathcal{T}^\dagger_{\nu_1,\mathbf{p}_1,\mathbf{K}_i;s_i,\mp,\pm}|v\rangle \times A_{\nu_1\mathbf{p}_1;\nu\mathbf{p}_i}(a_i,\mathbf{K}_i)\, \lambda^*_\nu, \quad (4.9)$$

in which we have used equation (3.54), which gives $(a-H)^{-1}$ acting on one e-X pair, while $a_i = \omega_p + \epsilon^{(e)}_{\mathbf{k}_i} + i\eta$. To go further, we have to differentiate the scalar product of e-X states with photocreated and initial electrons having the same spin or not, see equation (3.33).

4.3 Photocreated electron with spin different from the initial one

When the spins are different, the scalar product of e-X states, equation (4.9), differs from zero for $\nu_1 = \nu'$ and $\mathbf{p}_1 = \mathbf{p}_i$ only; so that

$$S_{\neq} = N_p \sum_{\nu',\nu} \lambda_{\nu'}\, A_{\nu'\mathbf{p}_i;\nu\mathbf{p}_i}(a_i,\mathbf{K}_i)\, \lambda^*_\nu. \quad (4.10)$$

Due to equation (3.58), the response function contains contributions from singlet and triplet trions ($S = 0, 1$) as expected, since with two different electron spins, we can construct the two types of trions. By using the expansion of $A_{\nu'\mathbf{p}';\nu\mathbf{p}}(a,\mathbf{K})$ in direct Coulomb scatterings, equation (3.55), we can expand this $S_{\neq}$ as

$$S_{\neq} = \sum_{n=0}^{+\infty} S^{(n)}_{\neq}, \quad (4.11)$$

where $S^{(n)}_{\neq}$ has n e-X scatterings $C^{\rm dir}_{\nu_1\mathbf{p}_1;\nu_2\mathbf{p}_2}$.

4.3.1 Zero order term in e-X interactions

The term without any e-X scattering,

$$S^{(0)}_{\neq} = N_p \sum_\nu \frac{|\lambda_\nu|^2}{a_i - E_{\nu\mathbf{p}_i\mathbf{K}_i}} = N_p \sum_\nu \lambda_\nu\, G_X(\omega_p,\mathbf{Q}_p;\nu)\, \lambda^*_\nu, \quad (4.12)$$

corresponds to the diagram of Figure 7a. This zero order term can be rewritten without any explicit reference to

exciton states ν, as

$$S_{\neq}^{(0)} = N_p |\lambda|^2 \Omega \left\langle \mathbf{r}=\mathbf{0} \left| \frac{1}{\omega_p - (h_X + \mathcal{E}_{\mathbf{Q}_p}^{(X)}) + i\eta} \right| \mathbf{r}=\mathbf{0} \right\rangle, \tag{4.13}$$

where h_X is the exciton relative motion Hamiltonian of equation (3.3).

4.3.2 First order term

The first order term in e-X scattering is zero since direct Coulomb processes impose non-zero momentum transfers, so that $C^{\rm dir}_{\nu' \mathbf{p}';\nu \mathbf{p}} = 0$ for $\mathbf{p}' = \mathbf{p}$ (see Eqs. (3.27–28)). This has to be contrasted with e-h diagrams for which a first order term exists. Note that this e-h diagram first order term is just a part of the ladder processes giving rise to the exciton propagator, so that it is in fact already included in the zero order exciton diagram of Figure 7a.

4.3.3 Second order term

Using equations (3.28, 55), the second order term in e-X scatterings reads

$$S_{\neq}^{(2)} = N_p \sum_{\nu',\nu} \lambda_{\nu'}\, G_X(\omega_p, \mathbf{Q}_p; \nu') \times \left[\sum_{\nu_1, \mathbf{q}_1} \frac{W^{\nu' \nu_1}_{-\mathbf{q}_1} W^{\nu_1 \nu}_{\mathbf{q}_1}}{\Delta_{\nu_1, \mathbf{q}_1}} \right] G_X(\omega_p, \mathbf{Q}_p; \nu)\, \lambda_{\nu}^*, \tag{4.14}$$

in which we have set

$$\Delta_{\nu, \mathbf{q}} = \omega_p - \left(E^{(X)}_{\nu, \mathbf{Q}_p + \mathbf{q}} + \epsilon^{(e)}_{\mathbf{k}_i - \mathbf{q}} - \epsilon^{(e)}_{\mathbf{k}_i} \right) + i\eta. \tag{4.15}$$

It is easy to check that the bracket of equation (4.14) can also be written as

$$\sum_{\nu_1, \mathbf{q}_1} \int \frac{i d\omega_1}{2\pi} W^{\nu' \nu_1}_{-\mathbf{q}_1}\, B(\omega_1, \mathbf{q}_1) \times G_X(\omega_p + \omega_1, \mathbf{Q}_p + \mathbf{q}_1; \nu_1)\, W^{\nu_1 \nu}_{\mathbf{q}_1}, \tag{4.16}$$

where $B(\omega_1, \mathbf{q}_1)$ is the standard bubble for the excitation of one e-h pair in the "Fermi sea", here reduced to the $(\mathbf{k}_i, s_i)$ electron,

$$B(\omega_1, \mathbf{q}_1) = (-1) \sum_{\mathbf{k}} \int \frac{i d\omega}{2\pi}\, g_i^{(e)}(\omega, \mathbf{k})\, g_i^{(e)}(\omega - \omega_1, \mathbf{k} - \mathbf{q}_1). \tag{4.17}$$

where $g_i^{(e)}(\omega, \mathbf{k})$ is the usual electron propagator, equal to $(\omega - \epsilon^{(e)}_{\mathbf{k}} + i\eta S_{\mathbf{k}, \mathbf{k}_i})^{-1}$, with $S_{\mathbf{k}, \mathbf{k}_i} = +1$ if $\mathbf{k} \neq \mathbf{k}_i$ and $S_{\mathbf{k}, \mathbf{k}_i} = -1$ if $\mathbf{k} = \mathbf{k}_i$. This second order term thus corresponds to the diagram of Figure 7b, with the e-X direct scattering vertex given by equation (3.27). It can also be rewritten without any reference to exciton states ν, as

$$S_{\neq}^{(2)} = N_p |\lambda|^2 \Omega \left\langle \mathbf{r}=\mathbf{0} \left| \frac{1}{\omega_p - (h_X + \mathcal{E}^{(X)}_{\mathbf{Q}_p}) + i\eta} \times \left[\sum_{\mathbf{q}_1} w_{\mathbf{q}_1}(\mathbf{r}) \frac{1}{\omega_p - (h_X + \mathcal{E}^{(X)}_{\mathbf{Q}_p + \mathbf{q}_1} + \epsilon^{(e)}_{\mathbf{k}_i - \mathbf{q}_1} - \epsilon^{(e)}_{\mathbf{k}_i}) + i\eta} \times w_{-\mathbf{q}_1}(\mathbf{r}) \right] \times \frac{1}{\omega_p - (h_X + \mathcal{E}^{(X)}_{\mathbf{Q}_p}) + i\eta} \right| \mathbf{r}=\mathbf{0} \right\rangle. \tag{4.18}$$

This unique e-X diagram (7b) has to be contrasted to the 6 diagrams corresponding to the second order term in $V_{\mathbf{q}}$ of the standard e-h procedure (the second order ladder diagram of Fig. 1c plus the 5 diagrams of Fig. 2). Let us again stress that the e-X and e-h many-body procedures are not strictly equivalent: Diagram (7b) contains terms in $V_{\mathbf{q}}^{(n)}$ with $n > 2$, not included in Figure 2: They are somehow "hidden" in the exciton propagator G_X which contains all ladder processes between the photocreated electron and the hole.

4.3.4 Third order term

In the same way, the third order term in e-X interaction reads

$$S_{\neq}^{(3)} = N_p \sum_{\nu',\nu} \lambda_{\nu'}\, G_X(\omega_p, \mathbf{Q}_p; \nu') \times \left[\sum_{\nu_1, \nu_2; \mathbf{q}_1, \mathbf{q}_2} \frac{W^{\nu' \nu_2}_{-\mathbf{q}_2} W^{\nu_2 \nu_1}_{\mathbf{q}_2 - \mathbf{q}_1} W^{\nu_1 \nu}_{\mathbf{q}_1}}{\Delta_{\nu_2, \mathbf{q}_2}\, \Delta_{\nu_1, \mathbf{q}_1}} \right] \times G_X(\omega_p, \mathbf{Q}_p; \nu)\, \lambda_{\nu}^*. \tag{4.19}$$

Using the exciton-photon vertex, the e-X direct scattering and the exciton propagator defined above, as well as the $(\omega, \mathbf{q})$ conservations standard for diagrams, it is easy to show that this third order term just corresponds to the diagram of Figure 7c. This unique diagram has again to be contrasted with the 21 e-h Feynman diagrams, at third order in Coulomb interaction (the third order ladder diagram of Fig. 1d plus the 20 diagrams of Fig. 3): This again shows that our e-X diagrams are far more convenient than the usual e-h Feynman diagrams, for this simple trion problem already. In more complicated problems, they should be even better. Using them, we can thus have some hope to calculate quantities which may appear as impossible to obtain from the usual e-h many-body procedure. Before going further, we can note that this third

order term also reads

$$S_{\neq}^{(3)} = N_p|\lambda|^2\Omega \left\langle \mathbf{r}=\mathbf{0}\left| \frac{1}{\omega_p - \left(h_X + \mathcal{E}_{\mathbf{Q}_p}^{(X)}\right) + i\eta} \right.\right.$$
$$\times \left[\sum_{\mathbf{q}_1,\mathbf{q}_2} w_{\mathbf{q}_2}(\mathbf{r}) \frac{1}{\omega_p - \left(h_X + \mathcal{E}_{\mathbf{Q}_p+\mathbf{q}_2}^{(X)} + \epsilon_{\mathbf{k}_i-\mathbf{q}_2}^{(e)} - \epsilon_{\mathbf{k}_i}^{(e)}\right) + i\eta} \right.$$
$$\times w_{\mathbf{q}_1-\mathbf{q}_2}(\mathbf{r}) \frac{1}{\omega_p - \left(h_X + \mathcal{E}_{\mathbf{Q}_p+\mathbf{q}_1}^{(X)} + \epsilon_{\mathbf{k}_i-\mathbf{q}_1}^{(e)} - \epsilon_{\mathbf{k}_i}^{(e)}\right) + i\eta}$$
$$\left.\left. \times w_{-\mathbf{q}_1}(\mathbf{r}) \right] \times \frac{1}{\omega_p - \left(h_X + \mathcal{E}_{\mathbf{Q}_p}^{(X)}\right) + i\eta} \right| \mathbf{r}=\mathbf{0} \right\rangle. \quad (4.20)$$

4.3.5 Higher order terms

While we did not even dare to draw the e-h Feynman diagrams at fourth order in Coulomb interaction, the simplicity of the e-X diagram procedure may lead us to think that the fourth order terms in e-X interactions should be given by the ladder diagram of Figure 7d. And similarly at higher orders. The correct result is actually somewhat more subtle.

The standard diagrammatic procedure with electron propagators, that we partly use here, as in the bubble $B(\omega_1, \mathbf{q}_1)$ of equation (4.17), is actually designed to describe *excitations* of the Fermi sea, i.e., processes in which $\mathbf{k} \neq \mathbf{k} - \mathbf{q}_n$. While at first and second order in e-X Coulomb processes, this is automatically fulfilled due to the $\mathbf{q} \neq \mathbf{0}$ constraint on the $W_{\nu'\nu}(\mathbf{q})$ scatterings, this is not imposed anymore at higher orders. Indeed, while in the fourth order term of $A_{\nu'\mathbf{p}_i;\nu\mathbf{p}_i}(a_i, \mathbf{K}_i)$, the first and last scatterings still impose $\mathbf{q}_1 \neq \mathbf{0}$ and $\mathbf{q}_3 \neq \mathbf{0}$, the intermediate scatterings simply impose $(\mathbf{q}_2 - \mathbf{q}_1) \neq \mathbf{0}$ and $(\mathbf{q}_3 - \mathbf{q}_2) \neq \mathbf{0}$ so that we can have $\mathbf{q}_2 = \mathbf{0}$. The precise calculation of the ladder diagram of Figure 7d confirms that this diagram only has excited Fermi sea pairs $(\mathbf{k}, \mathbf{k} - \mathbf{q}_2)$ with $\mathbf{q}_2 \neq \mathbf{0}$: The $\mathbf{q}_2 = 0$ ones are missing. Since the expansion of $A_{\nu'\mathbf{p}_i;\nu\mathbf{p}_i}(a_i, \mathbf{K}_i)$ does contain all possible scatterings of the photocreated exciton, i.e., the $\mathbf{q}_n = \mathbf{0}$ scatterings too, *we must add* the diagram of Figure 7e to the one of Figure 7d, in order to have all the fourth order terms of $S_{\neq}$. We can then be tempted to separate this additional diagram (and the similar ones at higher orders) from the set of ladder diagrams with 0, 1, 2 scatterings between the exciton and the electron, in which the exciton is always in a state $\mathbf{Q} \neq \mathbf{Q}_p$. As explained in more details below, this idea turns out to be a bad one.

To conclude, we can say that, when the photocreated electron and the initial electron have different spins, the response function $S_{\neq}$ corresponds to all possible diagrams between one electron and one exciton, with the following characteristics:

(i) These diagrams have one exciton line only, going from right to left, since there is one deep hole only: the photocreated one.

(ii) The photocreated (virtual) exciton suffers various scatterings, without any electron exchange with the initial electron, because, for two electrons having different spins, the deep hole can only recombine with the photocreated electron.

(iii) These diagrams have a unique *conduction-hole* line, going from left to right: Since the initial state has one electron $(\mathbf{k}_i, s_i)$ only, the corresponding initial "Fermi sea" can have one hole only, with a well defined momentum, $\mathbf{k}_i$, so that this hole cannot scatter. With such an essentially empty "Fermi sea", the conduction electron (s_i) has, on the opposite, plenty of sites to scatter.

4.3.6 Summation of e-X ladder diagrams

The trion absorption response function $S_{\neq}$ for photocreated and initial electrons with different spins, corresponds to the set of e-X ladder diagrams of Figure 7. They contain direct Coulomb processes only, *the exciton being always made with the same electron.*

Using equations (3.58–59), these diagrams can be summed up in terms of the trion energies and wave functions, as

$$A_{\nu'\mathbf{p}_i;\nu\mathbf{p}_i}(a_i, \mathbf{K}_i) = \sum_S \sum_{\eta_S} \frac{\langle \nu', \mathbf{p}_i|\eta_S\rangle\langle\eta_S|\nu, \mathbf{p}_i\rangle}{a_i - E_{\eta_S,\mathbf{K}_i}^{(T)}}, \quad (4.21)$$

where $\mathbf{p}_i$ is the relative motion momentum of the e-X pair made of the initial electron and the photocreated exciton, defined in equation (4.7). According to equations (4.2, 10), this leads to

$$S_{\neq} = N_p|\lambda|^2\Omega \sum_S \sum_{\eta_S} \frac{|\langle \mathbf{r}=\mathbf{0}, \mathbf{p}_i|\eta_S\rangle|^2}{\omega_p - (E_{\eta_S,\mathbf{K}_i}^{(T)} - \epsilon_{\mathbf{k}_i}^{(e)}) + i\eta}, \quad (4.22)$$

in agreement with our previous work on trion absorption [19].

By using the trion relative motion Hamiltonian h_T, which is such that $(h_T - \varepsilon_{\eta_S}^{(T)})|\eta_S\rangle = 0$, equation (4.22) also reads

$$S_{\neq} = N_p|\lambda|^2\Omega$$
$$\times \left\langle \mathbf{r}=\mathbf{0}, \mathbf{p}_i \left| \frac{1}{\omega_p - (h_T + \mathcal{E}_{\mathbf{K}_i}^{(T)} - \epsilon_{\mathbf{k}_i}^{(e)}) + i\eta} \right| \mathbf{r}=\mathbf{0}, \mathbf{p}_i \right\rangle, \quad (4.23)$$

From the above result, it is easy to recover the various terms of $S_{\neq}$ given in equations (4.13, 18, 20): Since $h_T = h_T^{(0)} + w(\mathbf{r}, \mathbf{u})$, we get, from equation (4.23) and the expansion (3.62) of $1/(a' - h_T)$,

$$S_{\neq} = N_p|\lambda|^2\Omega \left\langle \mathbf{r}=\mathbf{0}, \mathbf{p}_i \left| \frac{1}{a_i' - h_T^{((0)}} \right.\right.$$
$$\left. \left. + \frac{1}{a_i' - h_T^{(0)}} w(\mathbf{r}, \mathbf{u}) \frac{1}{a_i' - h_T^{(0)}} + \cdots \right| \mathbf{r}=\mathbf{0}, \mathbf{p}_i \right\rangle, \quad (4.24)$$

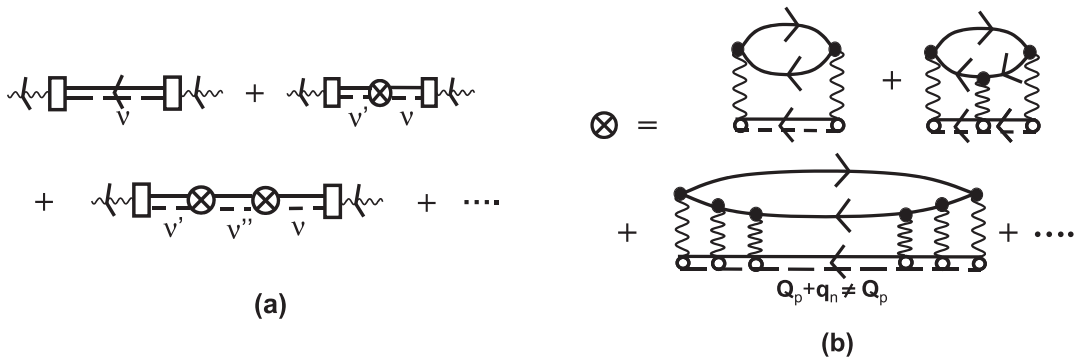

Fig. 8. (a) Integral equation (4.28) for the response function $S_{\neq}$ when the photocreated and initial electrons have different spins. The cross corresponds to all possible diagrams shown in Figure 8b. (b) Processes contributing to the exciton scattering $\Gamma_{\nu'\nu}$ appearing in the integral equation (4.28): In these "bubbles", the exciton momentum always differs from the initial momentum $\mathbf{Q}_p$, by construction.

where $a'_i = \omega_p + \epsilon^{(e)}_{\mathbf{k}_i} - \mathcal{E}^{(T)}_{\mathbf{K}_i} + i\eta$. By inserting the closure relation for free e-X states $|\nu, \mathbf{p}\rangle$ in front of each $1/(a'_i - h^{(0)}_T)$, the $S_{\neq}$ zero order term appears as proportional to

$$\left\langle \mathbf{r}=\mathbf{0}, \mathbf{p}_i \left| \frac{1}{a'_i - h^{(0)}_T} \right| \mathbf{r}=\mathbf{0}, \mathbf{p}_i \right\rangle = \left\langle \mathbf{r}=\mathbf{0} \left| \frac{1}{a'_i - h_X - \epsilon^{(eX)}_{\mathbf{p}_i}} \right| \mathbf{r}=\mathbf{0} \right\rangle, \quad (4.25)$$

which is nothing but the expectation value of $(\omega_p - h_X - \mathcal{E}^{(X)}_{\mathbf{Q}_p} + i\eta)^{-1}$ in the $|\mathbf{r}=\mathbf{0}\rangle$ state, in agreement with equation (4.13).

The first order term of $S_{\neq}$ is proportional to

$$\sum_{\nu',\mathbf{p}',\nu,\mathbf{p}} \left\langle \mathbf{r}=\mathbf{0}, \mathbf{p}_i \left| \frac{1}{a'_i - h^{(0)}_T} \right| \nu', \mathbf{p}' \right\rangle \langle \nu', \mathbf{p}' | w(\mathbf{r}, \mathbf{u}) | \nu, \mathbf{p} \rangle \times \left\langle \nu, \mathbf{p} \left| \frac{1}{a'_i - h^{(0)}_T} \right| \mathbf{r}=\mathbf{0}, \mathbf{p}_i \right\rangle, \quad (4.26)$$

which imposes $\mathbf{p}' = \mathbf{p}_i = \mathbf{p}$. It is thus equal to zero, since Coulomb scatterings impose non-zero momentum transfers.

In a similar way, the second order term is proportional to

$$\sum_{\nu',\mathbf{p}',\nu,\mathbf{p},\nu_1,\mathbf{p}_1,\nu_2,\mathbf{p}_2} \left\langle \mathbf{r}=\mathbf{0}, \mathbf{p}_i \left| \frac{1}{a'_i - h^{(0)}_T} \right| \nu', \mathbf{p}' \right\rangle \times \langle \nu', \mathbf{p}' | w(\mathbf{r}, \mathbf{u}) | \nu_2, \mathbf{p}_2 \rangle \left\langle \nu_2, \mathbf{p}_2 \left| \frac{1}{a'_i - h^{(0)}_T} \right| \nu_1, \mathbf{p}_1 \right\rangle \times \langle \nu_1, \mathbf{p}_1 | w(\mathbf{r}, \mathbf{u}) | \nu, \mathbf{p} \rangle \left\langle \nu, \mathbf{p} \left| \frac{1}{a'_i - h^{(0)}_T} \right| \mathbf{r}=\mathbf{0}, \mathbf{p}_i \right\rangle, \quad (4.27)$$

which imposes $\mathbf{p}' = \mathbf{p}_i = \mathbf{p}$ and $\mathbf{p}_2 = \mathbf{p}_1$. By writing $\mathbf{p}_1 = \mathbf{p}_i - \mathbf{q}_1$, it is straightforward to check that equation (4.27) leads to equation (4.18).

And so on, for the higher order terms.

4.3.7 An inappropriate separation

Let us end this part by explaining why it is inappropriate to treat separately the diagrams with excitons having a center of mass momentum $\mathbf{Q}_p + \mathbf{q}_n$ equal to $\mathbf{Q}_p$. This separation would lead to have $S_{\neq}$ as in Figure 8a, where the cross represents all the topologically connected e-X ladder diagrams of Figure 8b, with intermediate excitons having their center of mass momentum $\mathbf{Q}_p + \mathbf{q}_n \neq \mathbf{Q}_p$. So that

$$S_{\neq} = N_p \sum_{\nu',\nu} \lambda_{\nu'} \big[\delta_{\nu',\nu}\, G_X(\omega_p, \mathbf{Q}_p; \nu) + G_X(\omega_p, \mathbf{Q}_p; \nu')\, \Gamma_{\nu'\nu}\, G_X(\omega_p, \mathbf{Q}_p; \nu) + \cdots \big] \lambda^*_{\nu}. \quad (4.28)$$

Using the above section, the exciton scattering $\Gamma_{\nu'\nu}$ associated to the cross appears as $\langle \nu' | T(\omega_p, \mathbf{Q}_p, \mathbf{k}_i) | \nu \rangle$; so that all these ladder diagrams with $\mathbf{q}_n \neq \mathbf{0}$ can be summed up as

$$\left\langle \nu' \left| \frac{1}{\omega_p - h_X - \mathcal{E}^{(X)}_{\mathbf{Q}_p} + i\eta} + \frac{1}{\omega_p - h_X - \mathcal{E}^{(X)}_{\mathbf{Q}_p} + i\eta} \times T(\omega_p, \mathbf{Q}_p, \mathbf{k}_i) \frac{1}{\omega_p - h_X - \mathcal{E}^{(X)}_{\mathbf{Q}_p} + i\eta} + \cdots \right| \nu \right\rangle = \left\langle \nu' \left| \frac{1}{\omega_p - \tilde{h}_X - \mathcal{E}^{(X)}_{\mathbf{Q}_p} + i\eta} \right| \nu \right\rangle, \quad (4.29)$$

where $\tilde{h}_X = h_X + T(\omega_p, \mathbf{Q}_p, \mathbf{k}_i)$ can be seen as the Hamiltonian of the photocreated exciton dressed by its possible scatterings with the initial electron.

While physically appealing, this concept is technically useless: Indeed, e-X ladder processes can be summed up easily in terms of singlet and triplet trions if — and only if — *all* $\mathbf{q}_n$'s are included; the ones with $\mathbf{q}_n \neq \mathbf{0}$, as in $T(\omega_p, \mathbf{Q}_p, \mathbf{k}_i)$, are not known. Consequently, although physically nice, equation (4.29) which would lead to

$$S_{\neq} = N_p |\lambda|^2 \Omega \left\langle \mathbf{r}=\mathbf{0} \left| \frac{1}{\omega_p - \tilde{h}_X - \mathcal{E}^{(X)}_{\mathbf{Q}_p} + i\eta} \right| \mathbf{r}=\mathbf{0} \right\rangle, \tag{4.30}$$

has no practical use.

4.3.8 Conclusion

The response function, in the case of a photocreated electron with spin different from the initial one, is very similar to the response function of an exciton: It just contains e-X ladder processes. However, while, in the exciton case, there is no way to cut these ladder diagrams into separate pieces, in the trion case, the exciton — which plays the role of the hole in the e-X ladder processes — can possibly return into a $(\nu_n, \mathbf{Q}_p)$ state after a set of scatterings. This makes the corresponding diagram topologically separable. These two types of diagrams have however to be included in order to find all terms appearing in the response function (see Figs. 7d, e). Another important point is the fact that there is no additional diagram coming from possible electron exchanges, although, at any stage, the exciton can be made either with the photocreated or the initial electron. This is due to the fact that, in a well having electrons with different spins, the hole can only recombine with the photocreated electron; so that the number of exchanges which can take place before recombination, has to be even. Since from equation (3.23), two exchanges reduce to an identity, two exchanges are identical to no exchange at all.

A last comment: For most people, "trion" in fact means "ground state trion". Such trion corresponds to a singlet state, i.e., a state with two electrons having different spins: It thus corresponds to the e-X ladder diagrams of this paragraph. The real challenge with trions seen as interacting e-X pairs, in fact arises when the two electrons have the same spin, i.e., when the hole can recombine with any of the two electrons, for the electron exchange to explicitly enter the problem. For completeness, let us end this work by considering these triplet trions, although they are usually not the interesting ones.

4.4 Photocreated electron with spin identical to the initial one

Most probably, trions have not up to now been treated as a set of interacting e-X pairs because exchange processes in forming the exciton have appeared as quite tricky to handle. Our commutation technique allows to take care of these carrier exchanges in a simple and transparent way.

The response function $S_{\neq}$ for different electron spins, equation (4.22), reads in terms of trion states $|\eta_S\rangle$ with both, $S=0$ and $S=1$, as reasonable since, with $|+-\rangle$ electrons, singlet and triplet trions can be formed. On the opposite, if the two electrons have the same spin, as $|++\rangle$, we can only form $(S=1)$ trions, so that only $(S=1)$ trions should enter the response function: Exchange processes, which take place when the two electron spins are identical, must somehow withdraw the $(S=0)$ trion contributions from the final result. Let us now see how they do it.

When the electron spins are the same, the scalar product of two e-X states, equation (4.9), has two terms instead of one (see Eq. (3.33)): Beside the $\nu_1 = \nu'$ and $\mathbf{p}_1 = \mathbf{p}_i$ term, there is also an exchange term, so that

$$S_{=} = N_p \sum_{\nu',\nu} \lambda_{\nu'} \Big[A_{\nu'\mathbf{p}_i;\nu\mathbf{p}_i}(a_i, \mathbf{K}_i) - \sum_{\nu_1,\mathbf{p}_1} L_{\nu'\mathbf{p}_i;\nu_1\mathbf{p}_1} A_{\nu_1\mathbf{p}_1;\nu\mathbf{p}_i}(a_i, \mathbf{K}_i) \Big] \lambda^*_{\nu}. \tag{4.31}$$

4.4.1 Response function in terms of trions

Since, from equations (3.19, 59),

$$\sum_{\nu_1,\mathbf{p}_1} L_{\nu'\mathbf{p}_i;\nu_1\mathbf{p}_1} A^{(S)}_{\nu_1\mathbf{p}_1;\nu\mathbf{p}_i} = (-1)^S A^{(S)}_{\nu'\mathbf{p}_i;\nu\mathbf{p}_i}, \tag{4.32}$$

the response function for identical electron spins in fact reads

$$S_{=} = 2N_p |\lambda|^2 \Omega \sum_{\eta_1} \frac{|\langle \eta_1 | \mathbf{r}=\mathbf{0}, \mathbf{p}_i\rangle|^2}{\omega_p - (E^{(T)}_{\eta_1,\mathbf{K}_i} - \epsilon^{(e)}_{\mathbf{k}_i}) + i\eta}, \tag{4.33}$$

so that it contains contributions from triplet trions only, as expected. Note that, due to this restriction, $S_{=}$ cannot be written in a compact form in terms of the trion relative motion Hamiltonian h_T, as for $S_{\neq}$ in equation (4.23).

4.4.2 Expansion in e-X diagrams

If we now come back to equation (4.31) and use the expression (3.55) of $A_{\nu'\mathbf{p}';\nu\mathbf{p}}(a, \mathbf{K})$ in terms of e-X scatterings, we find that, beside the set of direct ladder diagrams of $S_{\neq}$ which come from the first term of equation (4.31), there are additional exchange diagrams coming from the second term. These exchanges however appear through the $L_{\nu'\mathbf{p}_i;\nu_1\mathbf{p}_1}$ factor which takes place *"at the end"* of a set of direct e-X scatterings, just before the exciton recombination, as $A_{\nu_1\mathbf{p}_1;\nu\mathbf{p}_i}(a_i, \mathbf{K}_i)$ contains direct Coulomb scatterings only. From equation (4.31), we thus see that for a photocreated electron with the same spin as the initial electron, the response function reads

$$S_{=} = S_{\neq} - \hat{S}, \tag{4.34}$$

$$\hat{S} = N_p \sum_{\nu,\nu_1,\mathbf{p}_1} \hat{\lambda}_{\mathbf{p}_i;\nu_1,\mathbf{p}_1} A_{\nu_1\mathbf{p}_1;\nu\mathbf{p}_i}(a_i, \mathbf{K}_i) \lambda^*_{\nu}, \tag{4.35}$$

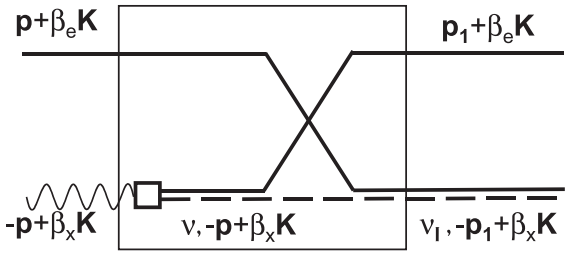

Fig. 9. Semiconductor-photon interaction $\hat{\lambda}_{\mathbf{p};\nu_1\mathbf{p}_1}$ dressed by the presence of one electron $\mathbf{k} = \mathbf{p} + \beta_X\mathbf{K}$ having the same spin as the photocreated electron.

where $\hat{S}$ comes from all possible exchange processes. $\hat{\lambda}_{\mathbf{p};\nu_1,\mathbf{p}_1}$, precisely given by

$$\hat{\lambda}_{\mathbf{p};\nu_1,\mathbf{p}_1} = \sum_{\nu} \lambda_\nu \, L_{\nu\mathbf{p};\nu_1\mathbf{p}_1} = \lambda \, \langle \mathbf{p} + \alpha_e\mathbf{p}_1 | \nu_1 \rangle, \quad (4.36)$$

appears as the exciton-photon vertex renormalized by carrier exchanges (see Fig. 9).

The zero order term of $\hat{S}$ in e-X Coulomb processes comes from the zero order term of $A_{\nu_1\mathbf{p}_1;\nu\mathbf{p}_i}(a_i, \mathbf{K}_i)$ in $C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}}$. Using equation (3.55), it reads

$$\begin{aligned}\hat{S}^{(0)} &= N_p \sum_{\nu} \frac{\hat{\lambda}_{\mathbf{p}_i;\nu,\mathbf{p}_i} \, \lambda^*_\nu}{a_i - E_{\nu\mathbf{p}_i\mathbf{K}_i}} \\ &= N_p \sum_{\nu} \hat{\lambda}_{\mathbf{p}_i;\nu,\mathbf{p}_i} \, G_X(\omega_p, \mathbf{Q}_p; \nu) \, \lambda^*_\nu. \end{aligned} \quad (4.37)$$

$\hat{S}^{(0)}$ thus corresponds to the diagram of Figure 10a, since

$$\hat{\lambda}_{\mathbf{p}_i;\nu,\mathbf{p}_i} = -\sum_{\mathbf{k}} \int \frac{id\omega}{2\pi} \, \hat{\lambda}_{\beta_X\mathbf{k}-\beta_e\mathbf{Q}_p;\nu,\beta_X\mathbf{k}-\beta_e\mathbf{Q}_p} \, g^{(e)}_i(\omega, \mathbf{k}). \quad (4.38)$$

$\hat{S}^{(0)}$ can also be written as

$$\begin{aligned}\hat{S}^{(0)} &= N_p |\lambda|^2 \sqrt{\Omega} \\ &\times \left\langle (1+\alpha_e)\mathbf{p}_i \left| \frac{1}{\omega_p - (h_X + \mathcal{E}^{(X)}_{\mathbf{Q}_p}) + i\eta} \right| \mathbf{r} = \mathbf{0} \right\rangle. \end{aligned} \quad (4.39)$$

As for the first order term, it reads

$$\begin{aligned}\hat{S}^{(1)} = \sum_{\nu} \left[\sum_{\nu_1,\mathbf{q}_1} \frac{\hat{\lambda}_{\mathbf{p}_i;\nu_1,\mathbf{p}_i-\mathbf{q}_1} \, W_{\nu_1\nu}(\mathbf{q}_1)}{\Delta_{\nu_1,\mathbf{q}_1}} \right] \\ \times G_X(\omega_p, \mathbf{Q}_p; \nu) \, \lambda^*_\nu. \end{aligned} \quad (4.40)$$

Note that, when the photocreated electron has a spin different from the initial one, the first order term of the response function $S_{\neq}$ is zero: The exciton-photon vertex then imposes $\mathbf{p} = \mathbf{p}_i$ while $C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}} = 0$ for $\mathbf{p}' = \mathbf{p}_i = \mathbf{p}$. On the opposite, first order e-X scatterings can exist when the two spins are identical, because the electron exchanges appearing in the dressed exciton-photon vertex do not impose $\mathbf{p}' = \mathbf{p}_i$ anymore. The first order term of $\hat{S}^{(1)}$ in fact corresponds to the diagram of Figure 10b, since, for $\mathbf{q}_1 \neq \mathbf{0}$,

$$\begin{aligned}\frac{\hat{\lambda}_{\mathbf{p}_i;\nu_1,\mathbf{p}_i-\mathbf{q}_1}}{\Delta_{\nu_1,\mathbf{q}_1}} &= -\sum_{\mathbf{k}} \int \frac{id\omega}{2\pi} \int \frac{id\omega_1}{2\pi} \\ &\times \hat{\lambda}_{\beta_X\mathbf{k}-\beta_e\mathbf{Q}_p;\nu_1,\beta_X\mathbf{k}-\beta_e\mathbf{Q}_p-\mathbf{q}_1} \\ &\times g^{(e)}_i(\omega, \mathbf{k}) \, g^{(e)}_i(\omega-\omega_1, \mathbf{k}-\mathbf{q}_1) \\ &\times G_X(\omega_p+\omega_1, \mathbf{Q}_p+\mathbf{q}_1; \nu_1). \end{aligned} \quad (4.41)$$

This first order term can also be written as

$$\begin{aligned}\hat{S}^{(1)} = N_p |\lambda|^2 \sqrt{\Omega} \sum_{\mathbf{q}_1} \Bigg\langle (1+\alpha_e)\mathbf{p}_i - \alpha_e\mathbf{q}_1 \Bigg| \\ \times \frac{1}{\omega_p - (h_X + \mathcal{E}^{(X)}_{\mathbf{Q}_p+\mathbf{q}_1} + \epsilon^{(e)}_{\mathbf{k}_i-\mathbf{q}_1} - \epsilon^{(e)}_{\mathbf{k}_i}) + i\eta} \\ \times w_{-\mathbf{q}_1}(\mathbf{r}) \, \frac{1}{\omega_p - (h_X + \mathcal{E}^{(X)}_{\mathbf{Q}_p}) + i\eta} \Bigg| \mathbf{r} = \mathbf{0} \Bigg\rangle. \end{aligned} \quad (4.42)$$

Using the same procedure, we can show that the second order terms of $\hat{S}$ in e-X Coulomb processes correspond to the diagram of Figure 10c *plus* the one of Figure 10d: here again, the e-X scatterings appearing in $A_{\nu'\mathbf{p}_i;\nu\mathbf{p}_i}(a_i, \mathbf{K}_i)$ impose $\mathbf{q}_n - \mathbf{q}_{n-1} \neq \mathbf{0}$ (with $\mathbf{q}_0 \equiv 0$): this leads to $\mathbf{q}_1 \neq \mathbf{0} \neq \mathbf{q}_2 - \mathbf{q}_1$, so that we can have both, $\mathbf{q}_2 \neq 0$ and $\mathbf{q}_2 = 0$. As the standard rules for calculating diagrams only lead to processes in which the Fermi sea is excited, the diagram of Figure 10c takes into account $\mathbf{q}_2 \neq 0$ excitations only. The $\mathbf{q}_2 = 0$ ones have thus to be included separately through the diagram of Figure 10d.

If we now turn to the third order terms of $\hat{S}$, they correspond to the diagrams of Figures 10e, f, g. And so on...

This thus shows that the additional exchange diagrams which take place when the photocreated and initial electrons have the same spin, and which are of crucial importance to withdraw the singlet contributions appearing in $S_{\neq}$, correspond to a set of "open" ladder diagrams which have the following characteristics:

(i) They all have one conduction-hole line only, without any hole scattering since the initial "Fermi sea" contains one electron only.

(ii) They all have an exciton possibly scattered by direct e-X processes without any exchange with the electron, although the two electrons have the same spin.

(iii) The fact that the exciton can be made with the initial or the photocreated electron appears, once and for all, in the renormalized exciton-photon interaction $\hat{\lambda}_{\mathbf{p}_i;\nu,\mathbf{p}}$, the photocreated hole possibly recombining with any of the two electrons. These diagrams thus start with a bare exciton-photon vertex λ^*_ν and end with a dressed one $\hat{\lambda}_{\mathbf{p}_i;\nu,\mathbf{p}}$ — or the reverse.

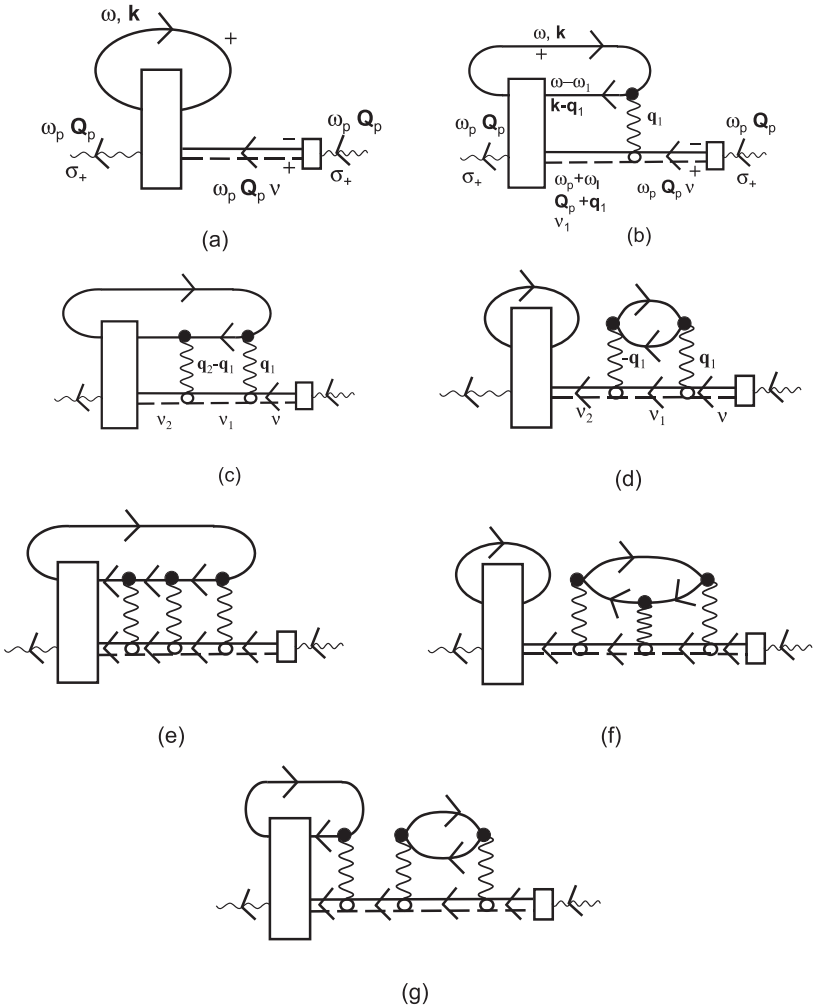

Fig. 10. Photon absorption with trion formation when the photocreated and initial electrons have the same spin. The diagrams of this figure have to be added to the diagrams of Figure 7. Their main effect is to withdraw the $S = 0$ trion contributions which cannot exist when the two electron spins are identical. (a) Zero order in e-X direct Coulomb scattering: The photocreated exciton already feels the presence of the initial electron through exchange processes included in the dressed semiconductor-photon interaction. (b) First order in $C^{\rm dir}_{\nu_1\mathbf{p}_1;\nu\mathbf{p}}$. (c) and (d) Second order. (e), (f) and (g) Third order.

(iv) Since the scattered exciton can be in a $(\nu_n, \mathbf{Q}_p)$ state, while the standard rules for diagrams lead to processes in which the "Fermi sea" — here the initial electron — is excited, we must add to the connected "open" ladder diagrams, diagrams with an "open" part separated from a ladder part, as in Figures 10d, f, g.

4.5 Volume dependence

From the response functions $S_{\neq}$ and $S_{=}$ in terms of trions given in equations (4.22, 33), we see that the trion oscillator strength reads in terms of the Fourier transform of the trion relative motion wave function $\langle \mathbf{r}, \mathbf{u}|\eta_S\rangle$, written with the "good" trion variables $\mathbf{r}$ and $\mathbf{u}$, defined in equations (3.2, 10). We also see that, when the photon polarization is such that only $(S = 1)$ trions can be formed, the oscillator strength of these $(S = 1)$ trions have an additional factor of 2, compared to the case in which $(S = 0, 1)$ trions can be created.

From the expression $|\lambda|^2\Omega|\langle \mathbf{r} = \mathbf{0}, \mathbf{p}_i|\eta\rangle|^2$ of the trion oscillator strength given in equation (4.22), it is easy to estimate its size for bound state (trion), partially dissociated state $(e+X)$ and totally dissociated state $(e+e+h)$. Let us recall it [19]:

(i) For bound trion, $\langle \mathbf{r}, \mathbf{u}|\eta\rangle$ has an extension over r of the order of a_X and an extension over u of the order of a_T, with $a_T \gg a_X$ since the trion binding energy is much weaker than the exciton one. In 2D, dimensional arguments lead to $\langle \mathbf{r} = \mathbf{0}, \mathbf{u} = \mathbf{0}|\eta\rangle \sim (a_X a_T)^{-1}$. From the spatial extension a_T of $\langle \mathbf{r} = \mathbf{0}, \mathbf{u}|\eta\rangle$, we then find, again from dimensional arguments, that, for p small, more precisely for $p \ll a_T^{-1}$, we have $\langle \mathbf{r} = \mathbf{0}, \mathbf{p}|\eta\rangle \sim \langle \mathbf{r} = \mathbf{0}, \mathbf{u} = \mathbf{0}|\eta\rangle\langle \mathbf{p}|\mathbf{u} \simeq \mathbf{0}\rangle a_T^2 \sim a_T/a_X L$, where L is the sample size $(L^2 = \Omega)$. This leads to a bound trion

oscillator strength $\sim |\lambda|^2 (a_T/a_X)^2$, independent from the sample volume.

(ii) For a partially dissociated trion, $\langle \mathbf{r}, \mathbf{u}|\eta\rangle$ now has an extension a_X over r but L over u, so that $\langle \mathbf{r}=\mathbf{0}, \mathbf{u}=\mathbf{0}|\eta\rangle \sim (a_X L)^{-1}$. For p small, we then have $\langle \mathbf{r}=\mathbf{0}, \mathbf{p}|\eta\rangle \sim 1/a_X$, so that partially dissociated trions have an oscillator strength $\sim |\lambda|^2 L^2/a_X^2$.

(iii) For totally dissociated trion, $\langle \mathbf{r}, \mathbf{u}|\eta\rangle$ has an extension L over r and L over u, so that $\langle \mathbf{r}=\mathbf{0}, \mathbf{u}=\mathbf{0}|\eta\rangle \sim L^{-2}$. For small p, this gives $\langle \mathbf{r}=\mathbf{0}, \mathbf{p}|\eta\rangle \sim 1/L$, which leads to an oscillator strength $|\lambda|^2$, which is sample volume independent, as the one of exciton diffusive states, i.e., free carriers. This oscillator strength is however smaller than the bound trion one, the trion extension a_T being somewhat larger than the exciton extension a_X.

The partially dissociated trion thus has essentially the same oscillator strength as the exciton, while the *bound trion oscillator strength* is a_T^2/L^2 smaller. It is thus *vanishingly small in the large sample limit*, i.e., the limit of solid state physics. This is after all not surprising: If we take a huge sample and if we add *just one* electron, we cannot expect any sizeable change in the photon absorption! From a technical point of view, if we consider the volume dependence of the diagrams representing the response function, we find that the dominant one, in the large L limit, is the bare exciton diagram of Figure 7a.

These quite simple arguments lead us to conclude that, when a line is seen, well below the exciton line, in the absorption spectrum of a macroscopic sample of doped semiconductor, it cannot be due to a (clean) trion because its oscillator strength would be a trion volume divided by a sample volume smaller than the exciton one. The observed line is most probably due to a trion having many-body effects with the $(N-1)$ other electrons of the semiconductor, or better to a photocreated exciton dressed by the N electrons already present in the sample. This quite difficult many-body problem will be addressed in a further work, using the tools we have established in Sections 3 and 4, which allow to treat excitons interacting with electrons through e-X diagrams.

5 Conclusion

We have considered the simplest problem on trion, namely the photon absorption in the presence of *one* electron. We have first shown that standard Feynman diagrams, with free electrons and holes, are totally inappropriate: For the trion being a bound state, Coulomb processes have to be included exactly, i.e., at all orders. While these Coulomb processes can be easily handled between the *two* carriers of an exciton, this is totally hopeless in the case of *three* carriers.

We propose to reduce this three-body problem to a two-body problem by considering the trion as an electron interacting with an exciton. Although physically appealing, this approach however faces the problem of the electron indistinguishability, i.e., the fact that the exciton can a priori be made with any of the two electrons. Our commutation technique, designed to deal with this problem, allows to overcome this difficulty.

We find that when the photocreated and initial electrons have different spins — which is what happens if we want to photocreate a ground state trion —, we can forget about these possible electron exchanges: The response function to the photon field just corresponds to a set of ladder diagrams between e-X pairs $(\nu, \mathbf{p}, \mathbf{K})$, with $\mathbf{K}$ being the center of mass momentum of the pair — constant in these ladder processes —, $\mathbf{p}$ the relative motion momentum of the e-X pair and ν characterizing the exciton relative motion level. The e-X interaction vertex $C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}}$ of these novel diagrams corresponds to *direct* Coulomb processes between electron and exciton, the "in" exciton ν and the "out" exciton ν' being made with the *same* electron. $C^{\rm dir}_{\nu'\mathbf{p}';\nu\mathbf{p}}$ reads in terms of the Fourier transform $w_{\mathbf{q}}(\mathbf{r})$ of the Coulomb potential $w(\mathbf{r}, \mathbf{u})$ between the electron e' and the exciton made of (e, h).

The possible electron exchanges are only important when the spins of the two electrons are identical, i.e., when triplet trions ($S_z = \pm 1$) are the only ones photocreated. We can include these exchanges through a dressed exciton-photon interaction. They thus play a role, once and for all, when the hole recombines, so that we are again left with direct e-X ladder processes only.

The physical reason for a so trivial consequence of carrier exchanges lies in the fact that two exchanges reduce to an identity; so that, either we end with no exchange at all — as when the photocreated hole can only recombine with the photocreated electron, which is what happens when the two electron spins are different —, or we end with zero or one exchange — as when the hole can recombine with either the photocreated electron or the initial electron, which is what happens when the two electrons have the same spin.

In Section 3 of this paper, we have also collected all important results on trions derived in our previous works — plus some unpublished ones. This "background on trion", which leads to this novel many-body procedure in terms of electrons and excitons, will be of great help to study the interaction of trion with carriers in doped semiconductors: Indeed, the existing literature on this very difficult many-body problem, which relies on standard electron-hole procedure — the only one at hand up to now —, is rather unsatisfactory, as explained in reference [19].

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